Concealed Carry

S. Andrews **R. Colacito** **M. M. Croce** **F. Gavazzoni***

Abstract

The slope carry consists of taking a long (short) position in the long-term bonds of countries with steeper (flatter) yield curves. The traditional carry is a long (short) position in countries with high (low) short-term rates. We document that: (i) the slope carry risk premium is slightly negative (strongly positive) in the pre (post) 2008 period, whereas it is *concealed* over longer samples; (ii) the traditional carry risk premium is lower post-2008; and (iii) there has been a sharp decline in expected global growth and global inflation post-2008. We connect these empirical findings through an equilibrium model in which investors price news shocks, financial markets are complete, and countries feature heterogeneous exposure to news shocks about both global output expected growth and global inflation.

JEL classification: F31; G12.
This draft: August 22, 2021.

*Andrews is affiliated with the Kenan-Flagler Business School at the University of North Carolina–Chapel Hill. Colacito is affiliated with the Kenan-Flagler Business School at the University of North Carolina–Chapel Hill and with NBER. Croce is affiliated with Bocconi University, IGIER, and CEPR. Gavazzoni is affiliated with INSEAD. We thank P. Bacchetta for early feedback and Biao Yang (Bocconi PhD program) for his excellent job as research assistant. We thank our discussants: Nancy Xu, Lukas Kremens, and Rob Richmond. We thank the participants to the Macro-Finance Triangle Seminars, the WFA, the EFA, the EEA, the Vienna Symposium on Foreign Exchange Markets and the INSEAD Finance seminars. All errors remain our own.

1 Introduction

The international finance literature has proposed several currency strategies based on sorting the cross section of countries on various criteria. While the performance of these strategies over long periods of time is well-documented, their behavior over different sample periods is not. In this study, we provide novel empirical evidence that shows that after 2008 there are relevant changes in the returns of two popular carry trade strategies based on sorting countries on the level and on the slope of their yield curves. In addition, we document that (i) expected global inflation and output growth have also changed substantially post-2008, and (ii) countries feature relevant heterogenous exposure to news shocks about both expected global growth and expected global inflation. In the context of a news-based asset pricing model, combining these two sources of heterogeneity rationalizes our empirical findings on carry trades.

Specifically, we document that (i) a strategy which consists in taking a short (long) position in low (high) interest rate currencies (henceforth "traditional carry trade") has experienced a marked decline post-2008, and (ii) a strategy that is short (long) the long-term bonds of countries with flatter (steeper) yield curves for one month (henceforth "slope carry") has a slightly negative return before the global financial crisis and a strongly positive one in the more recent part of the sample. While the first finding can be easily explained with the widespread compression of short-term interest rates that has taken place since 2008, the second finding is more puzzling. This is because the standing view in the literature is that a strategy based on investments in the cross section of long-term sovereign bonds should yield a null excess return (see for example Lustig *et al.* (2019)).

Our point of departure is that the lack of profitability of this carry strategy is *con-*

cealed over a sample that goes back about 30 years, due to a combination of a slightly negative excess return during the first half of the sample, followed by a strong positive excess return in the second half of the sample.

We propose an explanation of these empirical findings in the context of an endowment economy in which: (i) investors have recursive preferences, (ii) financial markets are complete, (iii) the growth rate of consumption in each country features heterogeneous exposure to a global expected growth rate component, and (iv) inflation is characterized by a country-specific exposure to a global expected inflation component. In the interest of parsimony we abstract away from local news shocks.

The first three ingredients are needed to obtain a persistent and profitable traditional carry risk premium as shown in Colacito *et al.* (2018). The fourth one is the key driver of the slope carry. Indeed, in our model investing in the long-term bonds of high global inflation exposure countries earns a positive excess return. This is because their bonds are exposed to more nominal interest rate risk. Furthermore, in times of lower than average expected inflation, such as in the post-2008 sample period, countries with high exposure to global inflation tend to have lower interest rates and steeper yield curves. In figure 1, we provide suggestive evidence implying that similar dynamics took place also in the immediate aftermath of the 2020-21 pandemic crisis.

Equivalently, our model predicts that, post-2008, investing in the long-term bonds of countries with steep yield curves should be profitable, consistent with the empirical findings that we put forward in our empirical investigation. A similar argument can be used to argue that, at times of higher than average expected inflation, such as in the period prior to the global financial crisis, high expected inflation countries should have flatter yield curves, and the slope carry strategy should earn a negative excess return.

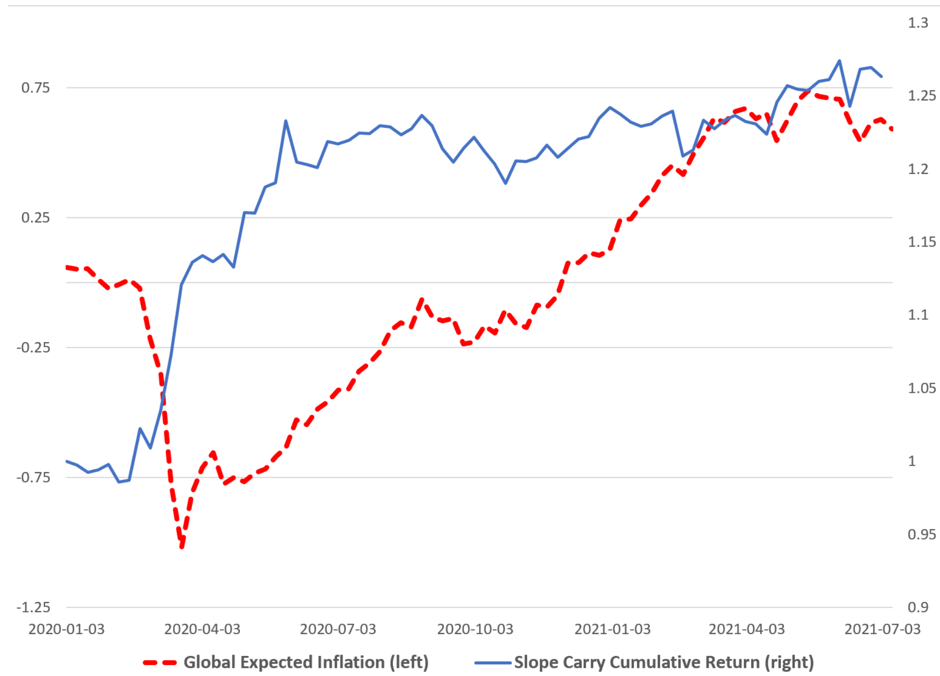


FIG. 1 - Global Inflation and Slope Carry Returns. The figure reports (i) the GDP-weighted average of the 5-year break-even inflation for the G10 countries that issue inflation indexed Treasuries (Australia, Canada, Germany, Japan, Sweden, UK, and US); and (ii) the cumulative return from the slope carry strategy applied G10 countries. The slope carry is short (long) the long-term bonds of countries with flatter (steeper) yield curves for one month.

Under our benchmark calibration, we are able to produce an average traditional carry annual spread of 3.11%, which declines when expected global growth and inflation are below their historical averages. This currency risk premium originates from a positive correlation between the returns to carry trade and expected global growth news. When, for example, a negative growth news shock hits, the carry trade yields a negative return due to the appreciation of the funding currencies (that is, countries with a high exposure to the growth rate of global GDP). Since our representative investors perceive this state of the world as negative, their marginal utility increases and a positive risk premium must be paid in equilibrium.

Furthermore, we document that our model features an unconditional slope carry excess return close to zero, which turns sharply positive during times of below-average expected global inflation. This result is primarily due to the interaction between the way in which countries are sorted and their inflation risk premium. According to the slope carry, we must take a long position in countries with relatively steeper yield curves. When expected inflation is below average, this sorting results in investing in countries with high exposure to inflation risk. Since investing in countries that load more on global expected inflation commands a larger risk premium, the slope carry must pay positive average returns. In our baseline calibration, the slope carry delivers an average excess return of 4.84% post-2008, in sharp contrast with its nearly zero unconditional average.

To discipline our calibration, we use OECD data on expected GDP growth rates and inflation for the 10 countries with the most traded currencies in the world. We construct measures of the global expected GDP growth rate and inflation as the cross-sectional average across all 10 countries' expectations. We document that countries display a substantial degree of heterogeneity in terms of their exposures to these global expectations by running regressions of each country's expected GDP growth rate and inflation on their global counterparts.

In particular, countries like Australia and New Zealand, which are commonly featured in the long leg of the traditional carry trade strategy, have very low exposures to the global expected GDP growth rate, whereas countries like Japan, which represent a typical funding currency in the traditional carry trade, feature a substantially higher degree of exposure to this source of risk. This confirms the findings of Colacito *et al.* (2018), which are obtained using the projection of GDP growth rates onto lagged values of price-dividend ratios.

We also find that countries such as the United Kingdom and Sweden have some of the largest exposures to global expected inflation. According to our model, the long-term bonds of these countries should command a substantial inflation risk premium, and their term structures of interest rates should be steeper (flatter) during times of below (above) average global expected inflation. This helps us rationalize our empirical findings concerning the slope carry excess return.

We complete our analysis by studying an extended version of our model featuring both an intertemporal elasticity greater than one and a global demand shock. In this setting, all of our main results are preserved and the global inflation news shocks explain a moderate share of the variance of the local yields, consistent with Duffee (2018).

Related literature. Our analysis relates currency risk and equilibrium exchange rates to macroeconomic factors and country-level characteristics (see, among others, Lustig *et al.* (2011, 2014), Bansal and Shaliastovich (2013), Lustig and Richmond (2019), Mueller *et al.* (2017), Sandulescu *et al.* (2020), and Zviadadze (2017)). Della Corte *et al.* (2009); Della Corte *et al.* (2011); Della Corte *et al.* (2016a) study the empirical behavior of spot and forward exhange rates. Hassan (2013), Hassan *et al.* (2015, 2016), Heyerdahl-Larsen (2015), Jiang (2019), Stathopoulos (2017), Richmond (2019), and Richmond and Jiang (2020) build equilibrium models of currency risk and relate them to country size, fiscal policy, habit formation, and trade network. Finally, Della Corte *et al.* (2016b), Koijen and Yogo (2019), Pavlova and Rigobon (2007, 2010, 2013), Lilley *et al.* (2020) study the equilibrium formation of exchange rates and how it relates to international capital flows.

On the one hand, we differ from prior studies for our attention to heterogenous exposure to global inflation risk and its implication for the concealed slope carry. On the other hand, our benchmark model with heterogeneous exposure to growth and inflation news is consistent with Verdelhan (2018), as it enables global long-run shocks to contribute to bilateral exchange rate variance. Our focus on large infrequent changes in global expected growth rate and inflation is related to work on rare disasters (Barro

(2006), Gabaix (2012), and Gourio (2012)) and its applications to international finance (see, for example, Gourio *et al.* (2014b), Farhi *et al.* (2015), and Chernov *et al.* (2018)).

Several articles have documented limitations of the long-run risks model in a one-country setting (see, for example, Le and Singleton (2010) and Beeler and Campbell (2012)). In our analysis, we document that while our complete-markets framework goes a long way in accounting for the international dynamics of asset prices and quantities, it does not fully replicate the cross section. Furthermore, our model abstracts away from country-specific news shocks, which may be relevant to obtain a more accurate matching of moments pertaining to the distribution of asset prices and quantities in the cross-section of countries.

Our paper also relates to the literature on inflation risk and its link to the real and nominal term structure of interest rates (see, among others, Piazzesi and Schneider (2005), Wachter (2006) Bansal and Shaliastovich (2013), Song (2017)). In our analysis, we document the presence of heterogenous exposure to global inflation risk and study its impact on the dynamics of currency risk premia.

The introduction of frictions (see, for example, Gabaix and Maggiori (2015), Maggiori (2017); Maggiori *et al.* (2020); Schreger and Du (2016);Froot and Stein (1991), Ready *et al.* (2017); Farhi and Werning (2014); Lustig and Verdelhan (2018); Zhang (2020); Du *et al.* (2020); Caballero *et al.* (2008); Gopinath *et al.* (2020); Kalemi-Ozcan *et al.* (2020); Avdjiev *et al.* (2020) and Bakshi *et al.* (2017)) may be important to (i) resolve these limitations, and (ii) address the empirical link with international capital flows (Froot and Ramadorai (2005), Gourinchas and Rey (2007), Gourio *et al.* (2014a), Coppola *et al.* (2020)).

Organization of the paper. The paper is organized as follows. Section 2 reports our empirical evidence concerning the heterogeneous exposure to global expected GDP growth and inflation in G10 countries. In Section 3 we present our economic model and its equilibrium conditions. Section 4 presents our main simulation results. Section 5 and Section 6 concludes.

2 Empirical Analysis

In this section, we show our main empirical results and introduce the moments that we replicate in our international macro-finance equilibrium model.

2.1 Preliminaries and Notation

Data. We obtain monthly zero coupon bond yield data for Australia, Canada, Germany, Japan, New Zealand, Norway, Sweden, Switzerland, United Kingdom and United States. In what follows, we will refer to these countries as G10 countries. When possible, all yields data are collected from Datastream for January 1995 through December 2018. The set of maturities for each country is reported in Appendix A.1. Exchange rates relative to the US Dollar are obtained from the Federal Reserve Economics Data (FRED) for the same sample period (see Appendix A.2).

We also collect data on the forecasts of real GDP growth and inflation for the same set of countries. The source for these data is the website of the Organisation for Economic Co-operation and Development (henceforth OECD). For GDP forecasts, the sample starts in 1961 for all countries, except for Germany (starting year: 1992), New Zealand (starting year: 1971), and Switzerland (starting year: 1966). For inflation forecasts, our sample starts in 1961 for all countries, except for Canada (starting year: 1993), Germany (starting year: 1996), and UK (starting year: 1991). A full description of the dataset is reported in Appendices A.3–A.4.

Notation. Let $P_{i,m,t}$ denote the price of a discount bond of maturity m in country i at date t. Let $R^h_{i,m,t}$ denote the date t gross holding period return associated to holding a bond of country i, maturity m for h periods, that is

$$R^h_{i,m,t} = \frac{P_{i,m-h,t}}{P_{i,m,t-h}}.$$

We shall denote as $r^h_{i,m,t}$ the logarithm of $R^h_{i,m,t}$. Let $E_{k,i,t}$ ($\Delta e_{k,i,t}$) denote the value (the natural logarithmic growth rate) of the currency of country i in units of the currency of country k at time t.

We denote as $RFX^n_{j,us,t}$ the one-month return on a strategy that is short the US 3-month bond and long the n-month bond of country j:

$$\log RFX^n_{j,us,t} = \left(r^1_{j,n,t} + \Delta e_{us,j,t}\right) - r^1_{us,3,t},$$

where $r^1_{j,n,t}$ is the date t 1-month log-return of investing in the n-month bond of country j, and $\Delta e_{us,j,t}$ is the date t log-growth rate of the exchange rate of currency j relative to the US Dollar.

2.2 Portfolio returns

Portfolios sorted on the yield curve level. At the beginning of each month, we sort countries based on the yield of their 3-month bond. Excluding the US, we group countries into three portfolios, where portfolio 1 (3) contains the three countries with the lowest (highest) level of the interest rate. We then compute the one-month return of a GDP-weighted portfolio that is short the US 3-month bond and long each of the

3-month bonds in each portfolio i:

$$\log RFX^{3}_{i,us,t} \quad = \quad \sum_{j \in i} w^{i}_{j,t} \cdot \log RFX^{3}_{j,us,t}, \quad \forall i \in \{1, 2, 3\}$$

where the weights are defined as $w^{i}_{j,t} = GDP_j / \left(\sum_{j \in i} GDP_j \right)$ and GDP_j is the average GDP of country j.

Portfolios sorted on the yield curve slope. Similarly, at the beginning of each month, we sort countries based on the spread between the 120-month (i.e. 10 year) yield and the 3-month yield (henceforth the slope of the yield curve). We then form portfolios by sorting countries on the slope of their yield curve, where portfolio 1 has flatter yield curves, and portfolio 3 has steeper yield curves. For each portfolio, we compute the one-month GDP-weighted return for the trading strategy that is short the US 3-month bond and is long each of the 120-month bonds in portfolio i for one month:

$$\log RFX^{120}_{i,us,t} \quad = \quad \sum_{j \in i} w^{i}_{j,t} \cdot \log \left(RFX^{120}_{j,us,t} \right), \quad \forall i \in \{1, 2, 3\}.$$

Currency Excess Returns. We report the average returns for portfolios sorted on the level and slope of the yield curve in Table 1 and Table 2, respectively. The top panel of Table 1 refers to the currency excess returns over our whole sample and shows that their average increases across interest rate-sorted portfolios as documented, among others, by Lustig *et al.* (2011). The excess return of a strategy that is long the high interest rate portfolio and short the low interest rate portfolio is around 5% and it is highly statistically significant. In what follows, we shall refer to this strategy as the "traditional carry".

These results stand in sharp contrast with the returns associated to the portfolios

TABLE 1: Traditional Carry

	1 (low)	2	3 (high)	3-1 (high-low)
Whole Sample				
Mean	−2.03	−0.05	2.90	4.93 [2.36]
Sharpe Ratio	−0.24	−0.01	0.28	0.48
Pre-08/2008				
Mean	−3.24	2.44	4.88	8.12 [3.25]
Sharpe Ratio	−0.36	0.35	0.67	0.89
Recurrent countries:	Jpn (100%) Swi (100%) Ger (41%)	UK (34%) Can (79%) Swe (60%)	Aus (90%) UK (66%) NZ (93%)	
Post-08/2008				
Mean	−0.47	−3.28	0.32	0.79 [0.23]
Sharpe Ratio	−0.06	−0.38	0.02	0.07
Recurrent countries:	Jpn (63%) Swi (94%) Ger (79%)	UK (94%) Can (71%) Swe (54%)	Aus (100%) NZ (100%) Nor (83%)	

Notes - The table reports the excess returns associated to borrowing at the 3 months interest rate of the US and investing in 3 months bonds of a GDP-weighted portfolio of countries with low (1), medium (2), and high (3) interest rates. The column label "3-1" reports the average return from being long portfolio 3 and short portfolio 1. Portfolios are rebalanced every month. Returns are in gross units. The analysis is conducted over three samples: 1/1995-12/2018 ("Whole sample"), 1/1995-7/2008 ("Pre-08/2008"), and 8/2008-12/2018 ("Post-08/2008"). Numbers in square brackets denote t-statistics.

sorted on the slope of the yield curve. Indeed, in the top panel of Table 2 we show that the currency excess returns across the three portfolios are very similar. As a result, a strategy that is long the portfolio of currencies with steeper yield curves and short the portfolio of currencies with flatter yield curves earns an excess return that is not statistically different from zero. This confirms the findings of Lustig *et al.* (2019). Since the sorting of this portfolio strategy is based on the slope of the term structure of interest rates, in what follows we will refer to this strategy as the "slope carry".

The relevance of subsamples. We further investigate these results by analyzing our pre- and post-August 2008 sub-samples. In Appendix C, we demonstrate that our results are robust to the specific choice of the sub-periods, that is, breaking our sample in August 2008 is not critical for our findings.

The results reported in the mid and bottom panels of Table 1 confirm the presence of a profitable traditional carry trade strategy both before and after our break. However, this excess return is sizeably smaller in the second part of the sample, a finding that is consistent with the sharp decline in interest rates in the aftermath of the global financial crisis. A visual inspection of the most recurrent currencies in each portfolio reveals a strong degree of similarity across the two sub-samples, with Japan and Australia typically appearing in the extreme portfolios for this strategy.

We find very different results when we focus on portfolios of countries sorted according to their yield curve slope. In the first part of the sample, the average currency excess returns in portfolios 1 to 3 are very similar to each other (see mid-panel of table 2). Hence a high-low investment strategy results in a excess return which is very close to zero (-13 basis points). The picture changes dramatically in the later part of the sample: in this period, a high-low strategy delivers a positive average excess return of 617 basis points (bottom panel of table 2). Equivalently, the null excess return in the full sample is the compositional outcome of offsetting excess returns in the two sub-samples.

Focusing on composition of these portfolios, we note that Australia is typically associated with portfolio 1, as it is consistently one of the countries with a flat yield curve, whereas Japan and UK switch between the two extreme portfolios pre- and post-break. Namely, UK (Japan) used to be a flatter- (steeper-) yield curve country pre-break and then it became a steeper- (flatter-) yield curve country post-break. Since these switches are not present when we form portfolios according to the level of the yield curve, they represent an important phenomenon that we take seriously and that we rationalize in the next section by looking at heterogeneous exposure to expected inflation.

Robustness. In Appendix C we conduct a series of robustness checks for our empiri-

TABLE 2: Slope Carry

	1 (flatter)	2	3 (steeper)	3-1 (steep-flat)
Whole Sample				
Mean	4.08	2.01	6.69	2.62
				[1.17]
Sharpe Ratio	0.38	0.20	0.66	0.24
Pre-08/2008				
Mean	6.42	3.67	6.29	−0.13
				[-0.04]
Sharpe Ratio	0.67	0.37	0.58	−0.01
Recurrent countries:	UK (75%)	Jpn (43%)	Jpn (55%)	
	Aus (72%)	Ger (56%)	Swe (58%)	
	NZ (76%)	Swi (40%)	Swi (51%)	
Post-08/2008				
Mean	1.05	−0.15	7.22	6.17
				[1.86]
Sharpe Ratio	0.09	−0.01	0.79	0.58
Recurrent countries:	Aus (75%)	Swi (58%)	UK (76%)	
	Jpn (78%)	Ger (45%)	Can (51%)	
	Nor (50%)	Can (45%)	Ger (55%)	

Notes - The table reports the excess returns associated to borrowing at the 3 months interest rate of the US and investing in the 10 year bonds of a GDP-weighted portfolio of countries with flatter (1), medium (2), and steeper (3) interest rates. The column label "3-1" reports the average return from being long portfolio 3 and short portfolio 1. Portfolios are rebalanced every month. Returns are in gross units. The analysis is conducted over three samples: 1/1995-7/2008 ("Whole sample"), 1/1995-7/2008 ("Pre-08/2008"), and 8/2008-12/2018 ("Post-08/2008"). Numbers in square brackets denote t-statistics.

cal evidence. Specifically we document that our main results are very similar to what reported in preceding sub-sections when (i) using log returns as opposed to gross returns, (ii) excluding the most extreme 10% of the distribution of returns, (iii) changing the base currency to Yen, Euro, and British Pound, (iv) changing the break-point to coincide with the end of calendar year 2007, and (v) using equal weights (as opposed to GDP weights) for the construction of the three portfolios.

2.3 Local and global expectations

Global expectations over time. We construct annual expectations for global inflation ($E_t\left[\pi_{G10,t+1}\right]$) and global real GDP growth ($E_t\left[\Delta y_{G10,t+1}\right]$) as the GDP-weighted cross-sectional averages across the 10 countries in our sample. When a country has a missing observation, we drop it for that year, and we rescale the GDP weights over the remaining countries.

Figure 2 reports the time series of global expectations over the same period that we used in our portfolio analysis. We note the following two important results. First, both GDP and inflation forecasts experienced a sizeable decline in 2009 in the aftermath of the global financial crisis. In theory, this fact is consistent with the realization of a negative long-run shock to global demand.

Second, if we split the sample into two parts, as we did in our portfolio analysis, the average inflation and GDP growth rate are lower post-break. Indeed the average inflation and real GDP growth forecasts are 1.92 and 2.74, respectively, in the period going from 1995 to 2007, and sharply decline to 1.59 and 1.42 in the sub-sample starting in 2008. The drop is present even if we remove 2009, i.e., the year of the sharpest decline for both forecasts.

Sensitivity of local expectations to global expectations. For each country in our cross section, we estimate the sensitivity of country-specific expected GDP growth and expected inflation with respect to their global counterparts. Specifically, we estimate the following regressions:

$$E_t\left[\Delta y_{i,t+1}\right] = \mu_{i,y} + \beta_{i,y} \cdot E_t\left[\Delta y_{G10,t+1}\right] + \varepsilon_{i,t} \quad (1)$$

$$E_t\left[\pi_{i,t+1}\right] = \mu_{i,\pi} + \beta_{i,\pi} \cdot E_t\left[\pi_{G10,t+1}\right] + \varepsilon_{i,t}, \quad (2)$$

for $i \in G10$ and where $E_t\left[\Delta y_{i,t+1}\right]$ and $E_t\left[\pi_{i,t+1}\right]$ denote the conditional expectations of GDP growth and inflation for each of the 10 countries, respectively. We conduct the estimations on the longest samples available (1961-2018 for GDP growth rate regressions; 1991-2018 for inflation regressions) and report our results in Table 3.

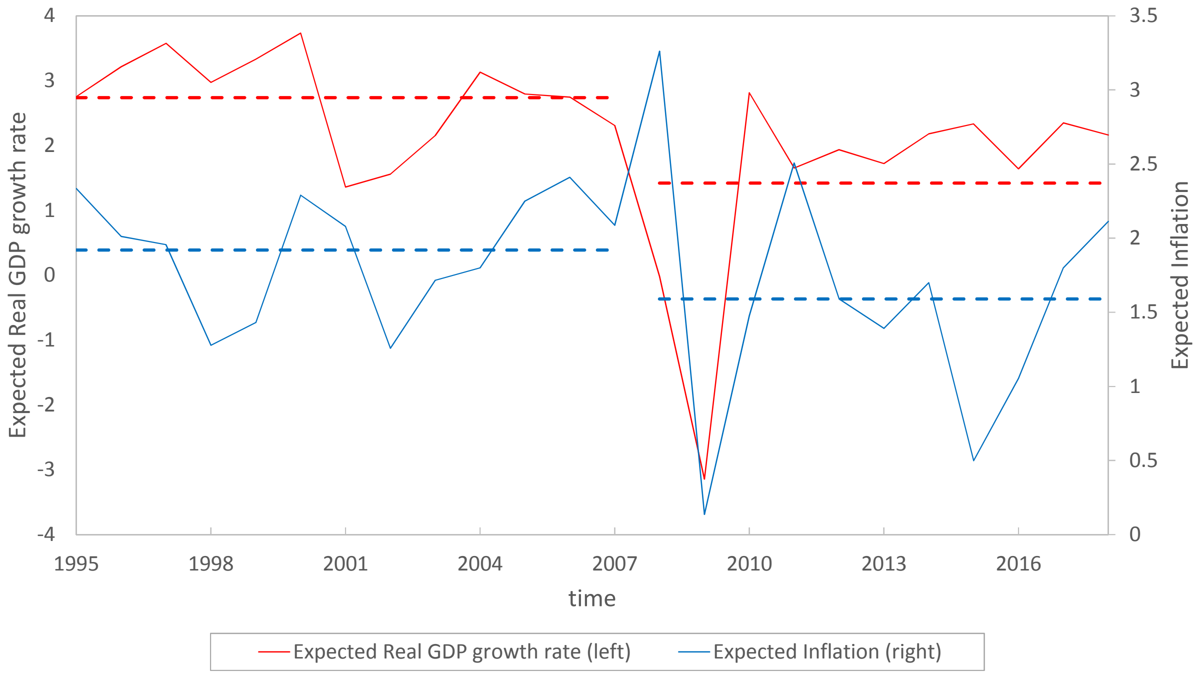


FIG. 2 - Expected Global Inflation and GDP growth rate. The figure reports expected global GDP growth rate ($E_t[\Delta y_{G10,t+1}]$) and inflation ($E_t[\pi_{G10,t+1}]$) computed as the cross-sectional GDP-weighted average across our G10 countries. The horizontal dashed lines represent the average expected inflation and GDP growth rate before and after 2008.

The estimates of the exposures to expected GDP growth document a substantial degree of heterogeneity in the cross section of countries. In particular, we note that countries' exposures to expected growth tend to line up with the typical sorting of countries according to the level of their respective yield curves. Indeed, Japan and Australia are at opposite sides of the spectrum in terms of their exposures to expected real growth. This result confirms the findings of Colacito *et al.* (2018), but it is obtained in a different way: we use expectations data as opposed to extracting expected global growth from global equity valuations.

In the bottom portion of Table 3, we also show the existence of a substantial degree of cross-sectional heterogeneity with respect to global expected inflation shocks, although the sorting of countries according to $\beta_{i,\pi}$ seems to be imperfectly correlated

TABLE 3: Expectations Exposures

Exposures to Expected GDP growth

	NZL	AUS	NOR	SWI	SWE	UK	US	CAN	GER	JPN
$\beta_{i,y}$	0.289	0.465	0.476	0.535	0.569	0.733	0.941	0.939	1.005	1.470
	(0.214)	(0.099)	(0.130)	(0.086)	(0.179)	(0.148)	(0.064)	(0.093)	(0.157)	(0.213)

Exposures to Expected Inflation

	NOR	AUS	NZL	CAN	JPN	GER	US	UK	SWI	SWE
$\beta_{i,\pi}$	0.204	0.531	0.583	0.598	0.693	0.794	1.108	1.234	1.444	1.811
	(0.125)	(0.130)	(0.231)	(0.087)	(0.202)	(0.048)	(0.086)	(0.293)	(0.250)	(0.255)

Notes - Exposures of each country's expected GDP growth rate and expected inflation to GDP weighted expectations of GDP and inflation. The first panel reports the estimates of $\beta_{i,y}$ in equation (1). The second panel reports the estimates of $\beta_{i,y}$ in equation (2). The numbers in parenthesis underneath each estimated coefficients are standard errors.

with the sorting according to $\beta_{i,y}$. Indeed, while Australia, New Zealand, and Norway are featured on the low end of the spectrum for both types of exposures, the sorting of the remaining countries appears to be more inverted. In Table A.1 of Appendix A.3 we replicate this exercise using alternative models to forecast inflation based on Stock and Watson (2008). The results are highly correlated with those reported in the bottom panel of Table 3.

In particular, we note that Japan (the country with the largest estimated exposure to real growth) has a relatively low inflation exposure, while the UK, Sweden, and Switzerland (which have a moderate real growth exposure) are the three countries with the largest inflation exposure. The imperfect link between exposure to global GDP growth and exposure to global inflation is relevant because it confirms that heterogeneous exposure to inflation news shocks is a distinct and novel dimension that can be relevant in understanding the cross section of currency returns.

We corroborate this point by studying the statistical significance of the differences in exposures of the three most recurrent countries in the extreme portfolios formed for our slope carry strategy. Specifically, we focus on Australia, Japan, and UK. Our results are reported in Table 4. We note that the exposure of expected GDP growth

TABLE 4: Differences of Expectations Exposures

	Expected GDP growth				*Expected Inflation*		
	AUS	**JPN**	**UK**		**AUS**	**JPN**	**UK**
AUS	–	1.005^{***}	0.268	**AUS**	–	0.161	0.703^{**}
		(0.211)	(0.192)			(0.299)	(0.334)
JPN		–	-0.737^{**}	**JPN**		–	0.541^{**}
			(0.299)				(0.270)
UK			–	**UK**			–

Notes - Differences of exposures of expected GDP growth rate (left) and expected inflation (right) between Australia, Japan, and UK. Each entry represents the difference between the exposures of the country in each column and the country in the row. Numbers in parenthesis are standard errors. One, two, and three stars represent statistical significance at the 10%, 5%, and 1% levels, respectively.

rates is larger for Japan compared to Australia and the United Kingdom (left panel). Indeed, a t-test for the null that $\beta_{Japan,y} = \beta_{Australia,y}$ and that $\beta_{Japan,y} = \beta_{UK,y}$ yields t-statistics equal to 4.76 and 2.46, respectively. When we repeat the same exercise for expected inflation, we note that the ranking of countries' sensitivities is different (right panel). Specifically, the sensitivity of the UK's expected inflation is the largest; a t-test for the null that $\beta_{UK,\pi} = \beta_{Australia,\pi}$ and that $\beta_{UK,\pi} = \beta_{Japan,\pi}$ yields t-statistics equal to 2.10 and 2.00, respectively.

In our theoretical model, we explain the connection between these estimated exposures to expected GDP growth and inflation and the risk-premia on the traditional and slope carries. In particular, we document that the excess return on the traditional carry reflects exposures to expected GDP growth (β_y), while the excess return on the slope carry is primarily determined by exposures to expected inflation (β_π).

Since the composition of the traditional carry portfolios have remained largely unchanged before and after the break, the traditional carry risk premium is a reflection of nearly unchanged portfolio-level exposures to expected growth and inflation news shocks. This explains why the excess returns on the traditional carry have remained positive across the two regimes.

Conversely, the large swing in expected global inflation and growth that we observe post-break is associated with a large redistribution of countries across our slope-sorted portfolios. This compositional change has caused a drastic change in the portfolio-level exposures of the top and bottom portfolios of the slope carry to growth and inflation risk. Specifically, the UK has moved from portfolio 1 to portfolio 3, and Japan has moved in the opposite direction. Through the lens of our model, inflation risk has a positive market price of risk and hence the post-break reallocation of high-β_π (low-β_π) countries to portfolio 3 (1) causes the slope carry to earn a positive risk premium.

3 The Model

In this section we present an equilibrium model that can explain our empirical findings by taking into account the documented heterogeneous exposure to global real growth and inflation. While our model abstracts away from endogenous trade in the consumption goods market (Colacito *et al.* (2018)), it constitutes a useful benchmark in the international finance literature, and it has been applied to the analysis of exchange rate volatility (Colacito and Croce (2011a)), international term structure of interest rates (Bansal and Shaliastovich (2013)), and gravity in exchange rate fluctuations (Lustig and Richmond, 2019), among others. We follow the literature and focus on this setup due to its ability to deliver closed-form solutions for all the objects of interest, and leave a fully fledged general equilibrium analysis to future research.

3.1 Setting

Preferences. The economy consists of N countries, indexed by $i \in \{1, 2, \ldots, N\}$. Each country is populated by a representative agent with recursive preferences:

$$U_{i,t} = (1-\delta)\log C_{i,t} + \delta\theta \log E_t \exp\left\{\frac{U_{i,t+1}}{\theta}\right\},$$

where γ denotes the risk aversion coefficient, δ is the subjective discount factor, and $\theta = 1/(1-\gamma)$. These preferences correspond to Epstein and Zin (1989b) preferences for the case of unit intertemporal elasticity of substitution (henceforth IES). Throughout our analysis, we will assume that $\gamma > 1$, which implies that $\theta < 0$. Under this assumption, news shocks are priced.

Real Consumption and Inflation. Let $x_{c,t}$ and $x_{\pi,t}$ denote time-varying components in expected global consumption growth and inflation, respectively. We model these components as follows:

$$\underbrace{\begin{bmatrix} x_{\pi,t} \\ x_{c,t} \end{bmatrix}}_{x_t} = \underbrace{\begin{bmatrix} \rho_\pi & 0 \\ \rho_{c\pi} & \rho_c \end{bmatrix}}_{K} \cdot \begin{bmatrix} x_{\pi,t-1} \\ x_{c,t-1} \end{bmatrix} + \underbrace{\begin{bmatrix} \sigma_{x,\pi} & 0 \\ 0 & \sigma_{x,c} \end{bmatrix}}_{\Sigma} \begin{bmatrix} \varepsilon_{\pi,t} \\ \varepsilon_{c,t} \end{bmatrix}, \tag{3}$$

in which $\varepsilon_{\pi,t}$ and $\varepsilon_{c,t}$ are iid $N(0,1)$ news shocks. Our specification allows expected inflation to be correlated with expected growth according to the coefficient $\rho_{c\pi}$. We can think of $\rho_{c\pi} < 0$ as capturing the relative dominance of global aggregate supply shocks relative to global demand shocks. The parameter ρ_c (ρ_π) determines the half-life of growth (inflation) news shocks.

At the country level, the log-growth rate of consumption is given by

$$\begin{aligned}\Delta c_{i,t+1} &= \mu_c^i + \beta_i^c x_{c,t} + \sigma_c \eta_{i,t+1}^c \\ \pi_{i,t+1} &= \mu_\pi^i + \beta_i^\pi x_{\pi,t} + \sigma_\pi \eta_{i,t+1}^\pi,\end{aligned}$$

where β_i^c and β_i^π capture country-specific heterogeneous exposure to news shocks about global consumption growth and inflation, and the shocks $\eta_{i,t+1}^c$ ($\eta_{i,t+1}^\pi$) are distributed as standard normals. These shocks represent short-run growth (inflation) risk and are independent within and across each country.

We detail our calibration in the next section. Here we note two points. First, we think of the base country in our cross section as having $\beta_i^c = \beta_i^\pi = 1$. Second, we allow for country-specific growth and inflation rates, μ_c^i and μ_π^i, in order to have a properly defined cross section of short-term risk free rates. This is an innocuous assumption that we could relax either by having country-specific discount rates δ^i or by modeling very persistent deviations from a common global stochastic trend (as in Colacito *et al.* (2018)).

Financial markets. We assume that there is a complete set of state and date contingent bonds that each investor has access to in frictionless financial markets at each point in time.

3.2 Equilibrium Pricing

In what follows, we report the analytical results that are essential to interpret the implications of our model. Detailed derivations are available in Appendices D and E.

Real SDF. Each country i has the following real stochastic discount factor:

$$m_{i,t+1}^{real} = \bar{m}_i^{real} - \beta_c^i x_{c,t} - k_{\varepsilon c}^i \sigma_{x,c} \varepsilon_{c,t+1} + k_{\varepsilon \pi}^i \sigma_{x,\pi} \varepsilon_{\pi,t+1} - \gamma \sigma_c \eta_{i,t+1}^c,$$

where the unconditional level of the real log-SDF is

$$\bar{m}_i^{real} = \log \delta - \frac{1}{2}(1-\gamma)^2 \sigma_c^2 - \mu_c^i - \frac{1}{2}\left[(k_{\varepsilon c}^i \sigma_{x,c})^2 + ((k_{\varepsilon \pi} \sigma_{x,\pi})^2\right],$$

and

$$\begin{aligned} \mu_c^i &= \overline{\mu}_c + \overline{\mu}_c(1-\beta_c^i), \\ k_{\varepsilon c}^i &= (\gamma - 1)\beta_c^i \left(\frac{\delta}{1-\delta\rho_c}\right) > 0, \\ k_{\varepsilon \pi}^i &= -\rho_{c\pi} k_{\varepsilon c}^i \left(\frac{\delta}{1-\delta\rho_\pi}\right). \end{aligned} \tag{4}$$

All of the heterogeneity across countries derives from their heterogeneous exposure to real growth news shocks, β_c^i. Real expected growth can change either because of changes in expected global growth ($\varepsilon_{c,t+1}$ shocks) or indirectly because of the effects of expected inflation on expected global growth ($\varepsilon_{\pi,t+1}$ shocks).

When $\rho_{c\pi} < 0$, news to global inflation and news to real growth determine movements of the stochastic discount factors in opposite directions. Indeed, the third equation in (4) shows that when $\rho_{c\pi} < 0$ the composite coefficient $k_{\varepsilon\pi}^i$ is larger than zero, thus implying that positive shocks to expected global inflation cause the marginal utility increase. The opposite occurs for global growth news shocks, that is, the representative agent marginal utility decreases when $\varepsilon_{c,t+1} > 0$. The market price of short-run growth shocks, $\eta_{i,t+1}^c$, is assumed to be constant across countries.

We model μ_c^i as decreasing in β_c^i so that country-specific unconditional average real risk-free rates,

$$\bar{r}^i = \mu_c^i - \log\delta - \left(\frac{1}{2} - \frac{1}{\theta}\right)\sigma_c^2,$$

are decreasing in β_c^i, holding everything else equal (see first equation in (4). This is a reduced form way to ensure that low real risk-free rate countries are also high-β_c^i countries, consistent with the analysis of Colacito *et al.* 2018.

Nominal SDF. In each country the nominal stochastic discount factor, $m_{i,t+1}$, is $m_{i,t+1}^{real} - \pi_{i,t+1}$. A as a result, we obtain:

$$m_{i,t+1} = \bar{m}_i - \beta_c^i x_{c,t} - \beta_\pi^i x_{\pi,t} - k_{\varepsilon c}^i \sigma_{x,c}\varepsilon_{c,t+1} + k_{\varepsilon\pi}^i \sigma_{x,\pi}\varepsilon_{\pi,t+1} - \gamma\sigma_c\eta_{i,t+1}^c - \sigma_\pi\eta_{i,t+1}^\pi, \tag{5}$$

where $\bar{m}_i = \bar{m}_i^{real} - \mu_\pi^i$ and where we specify

$$\mu_\pi^i = \overline{\mu}_\pi - \overline{\mu}_\pi(1 - \beta_\pi^i), \tag{6}$$

in order to make high-average inflation countries also high-β_π^i countries, as in our data. Even though agents in each country are heterogeneous with respect to global inflation news shocks, they are identical when it comes to pricing short-run inflation shocks ($\eta_{\pi,t+1}$). This assumption grants parsimony without loss of generality for our results. Given this log-linear representation of our SDF, our term structure inherits standard properties common to all affine log-normal models.

Exchange rates and decomposition of the nominal SDF. Since financial markets are assumed to be complete, the log-exchange rates between the currencies of any two countries i and j are given by the difference of their respective stochastic discount factors:

$$\Delta e_{ij,t+1} = m_{j,t+1} - m_{i,t+1}.$$

We analyze the properties of our currency strategies by decomposing the SDFs into a permanent and a transitory component, as in Chabi-Yo and Colacito (2019), Lustig *et al.* (2019), and Sandulescu *et al.* (2020). Specifically, we solve the eigenfunction problem of Alvarez and Jermann (2005) and Hansen (2012) to obtain a permanent and transitory component of the log-stochastic discount factor of each country such that:

$$m_{i,t+1} = m^P_{i,t+1} + m^T_{i,t+1}.$$

The permanent and the transitory components are

$$m^P_{i,t+1} \quad = \quad \bar{m}^P_i - \beta^i_c k^{i,P}_{\varepsilon c} \sigma_{xc} \varepsilon_{c,t+1} - \left(\frac{\beta^i_\pi}{1-\rho_\pi} - \beta^i_c k^{i,P}_{\varepsilon\pi} \right) \sigma_{x\pi} \varepsilon_{\pi,t+1} - \gamma \sigma_c \eta^c_{i,t+1} - \sigma_\pi \eta^\pi_{i,t+1},$$

and

$$m^T_{i,t+1} \quad = \quad \bar{m}^T_i - \beta^i_c x_{c,t} - \beta^i_\pi x_{\pi,t} + \frac{\beta^i_c}{1-\rho_c} \sigma_{xc} \varepsilon_{c,t+1} + \left(\beta^i_\pi + \beta^i_c \cdot \frac{\rho_{c\pi}}{1-\rho_c} \right) \frac{\sigma_{x\pi}}{1-\rho_\pi} \varepsilon_{\pi,t+1},$$

respectively, and the composite parameters are defined as

$$k^{i,P}_{\varepsilon c} = k^i_{\varepsilon c} + \frac{1}{1-\rho_c}, \qquad k^{i,P}_{\varepsilon\pi} = k^i_{\varepsilon\pi} + \frac{-\rho_{c\pi}}{(1-\rho_\pi)(1-\rho_c)}.$$

When $\rho_{c\pi} < 0$, tboth $k^{i,P}_{\varepsilon c}$ and $k^{i,P}_{\varepsilon\pi}$ are positive. The intercepts $\bar{m}^P_i$ and $\bar{m}^T_i$ are defined in Appendix D.

Let $P^n_{i,t}$ denote the price of a nominal bond with maturity n in country i at time t. We use $hpr^\infty_{i,t+1}$ to denote the holding period return of a zero-coupon bond with infinite maturity in country i:

$$hpr^\infty_{i,t+1} := \lim_{n\to\infty} \log \left(\frac{P^{n-1}_{i,t+1}}{P^n_{i,t}} \right).$$

As in Alvarez and Jermann (2005), the transitory component $m^T_{i,t+1}$ is equivalent to the negative of the logarithm of the holding period return on an infinite maturity bond:

$$m^T_{i,t+1} = -hpr^{\infty}_{i,t+1}.$$

This means that when an investor in country j invests in the infinite maturity bond of country i, the exchange rate acts as a perfect hedge against the risk associated with $hpr^{\infty}_{i,t+1}$ since

$$\Delta e_{ji,t+1} = m_{i,t+1} - m_{j,t+1} = m^P_{i,t+1} - hpr^{\infty}_{i,t+1} - m_{j,t+1}. \tag{7}$$

Equivalently, the risk premium associated to this strategy reflects only the exposure to the permanent component of the SDF of country i.

3.3 Traditional carry

Sorting countries into portfolios. In the traditional carry strategy, countries are sorted according to their relative short-term interest rates. In our model, the logarithm of the nominal risk-free rate in each country is

$$r^i_{1,t} \quad = \quad \bar{r}^i + \beta^i_{\pi} x_{\pi,t} + \beta^i_c x_{c,t}, \tag{8}$$

where

$$\bar{r}^i = \left(\mu^i_c + \mu^i_{\pi}\right) - \log\delta - \left(\frac{1}{2} - \frac{1}{\theta}\right)\sigma^2_c - \frac{1}{2}\sigma^2_{\pi}.$$

Hence, the sorting of our countries is driven by both country-specific fixed effects, $\bar{r}^i$, and by the interaction of country-specific exposures with expectations about global real growth and inflation, $\beta^i_{\pi} x_{\pi,t} + \beta^i_c x_{c,t}$. In the data, the sorting of countries according to their short-term interest rate is very stable over time. In order to replicate this empirical fact, we calibrate our model so that the unconditional averages of the risk-free rates, $\bar{r}^i$, tend to dominate the relative sorting of the risk-free rates.

More specifically, if we consider country i and j, the unconditional interest rate differential depends on

$$\bar{r}^i - \bar{r}^j = \bar{\mu}_\pi \left[(\beta_\pi^i - \beta_\pi^j) - \frac{\bar{\mu}_c}{\bar{\mu}_\pi}(\beta_c^i - \beta_c^j) \right]. \tag{9}$$

Since in the data $\frac{\bar{\mu}_c}{\bar{\mu}_\pi} \approx 2$, heterogeneity across β_c's is quantitatively more important than that in β_π's. Therefore under the ergodic distribution implied by our model, high-β_c countries are typically low-interest rate countries.

Traditional carry excess returns. Let us use the index b to denote the base country, and normalize the base country's exposure to global growth to one, $\beta_c^b = 1$. The expected excess return of a strategy that is short the risk-free rate of the base country and long the short-term rate of country i is:

$$\begin{aligned} \log E_t \left[RX_{i,t+1}^1 \right] &= \log E_t \exp \left\{ -r_{1,t}^b + r_{1,t}^i + \Delta e_{bi,t+1} \right\} \\ &= V_t \left[m_{t+1}^b \right] - cov_t \left[m_{t+1}^b, m_{t+1}^i \right] \\ &= V_t \left[m_{t+1}^b \right] - \beta_c^i (k_{\varepsilon c}^2 \sigma_{xc}^2 + k_{\varepsilon\pi}^2 \sigma_{x\pi}^2), \end{aligned} \tag{10}$$

where $k_{\varepsilon c} = (\gamma - 1)\left(\frac{\delta}{1-\delta\rho_c}\right)$ and $k_{\varepsilon\pi} = -\rho_{c\pi} k_{\varepsilon c}\left(\frac{\delta}{1-\delta\rho_\pi}\right)$. Equation (10) implies that all of the cross-sectional heterogeneity in risk premia is driven solely by β_c^i. In this case, β_π^i is irrelevant because news to global inflation are priced only through their disruptive effect on expected long-term growth (see $\beta_c^i k_{\epsilon\pi}$ in the equilibrium nominal SDF in equation (5)). Specifically, investing in high-β_c^i countries produces an insurance premium as the currency of the targeted country provides a hedge against adverse growth news shocks (Colacito *et al.* 2018).

Since a traditional carry strategy is long the currency of high-interest rate (H) countries (low-β_c countries, henceforth β_c^L) and short the currency of the low-interest rate (L) countries (high-β_c countries, henceforth β_c^H), the resulting traditional carry risk premium, $E[carry^T]$, is:

$$E[carry^T] := \log E_t\left[RX^1_{\mathbf{H},t+1}\right] - \log E_t\left[RX^1_{\mathbf{L},t+1}\right] \tag{11}$$
$$= \left(\beta_c^H - \beta_c^L\right)\left[k^2_{\varepsilon c}\sigma^2_{xc} + k^2_{\varepsilon\pi}\sigma^2_{x\pi}\right].$$

The expression for the traditional carry risk premium in (11) specializes the findings of Lustig *et al.* (2011) to the economy that we analyze in this paper. It confirms that the currency premium reflects heterogeneous exposure to a global risk factor in the cross section of countries. In the context of our economy, the relevant source of heterogeneity is associated to the exposure to global real growth news shocks. In the next section, we document how the heterogeneous exposure to expected inflation shocks enables our model to explain the cross section of slope carry excess returns.

3.4 Slope carry

Sorting countries into portfolios. Based on the slope carry strategy, we sort countries according to the slope of their term structure of yields. Our model is affine and it features two state variables comprised in the vector x_t (see equation (3)). As a result, the yield on an n-period maturity bond is:

$$r^n_{i,t} = A^n_i + B^{n'}_i \cdot x_t,$$

where the coefficients A^n_i and $B^{n'}_i$ are consistent with no-arbitrage and are detailed in appendix F.5.

We follow Lustig *et al.* (2019) and focus on the slope of the yield curve determined by the difference between the yields on the infinite maturity and on the one-period bonds in each country. By letting $n \to \infty$, it is possible to show that:

$$\begin{aligned}\lim_{n\to\infty} B_i^n &= [0 \quad 0] \\ \lim_{n\to\infty} A_i^n &= \bar{r}^i - \beta_i'(I-K)^{-1}\Sigma\left[\frac{\Sigma'}{2}\left[(I-K)^{-1}\right]'\beta_i + \Lambda_i\right],\end{aligned}$$

that is, the yield on the infinite maturity bond is constant and equal to $r_i^\infty = \lim_{n\to\infty} A_i^n$. Combining this result with the equilibrium risk-free rate in equation (8), we obtain the slope of the yield curve in each country:

$$slope_{i,t}^\infty = \overline{slope_i}^\infty - \beta_c^i x_{c,t} - \beta_\pi^i x_{\pi,t},$$

where $\overline{slope_i}^\infty = -\beta_i'(I-K)^{-1}\Sigma\left[\Sigma'\left[(I-K)^{-1}\right]'\beta_i/2 + \Lambda_i\right]$. The sorting of our countries is again driven by both country-specific fixed effects, $\overline{slope_i}^\infty$, and by the interaction of country-specific exposures with transitory fluctuations in the expectations about global real growth and inflation, $\beta_\pi^i x_{\pi,t} + \beta_c^i x_{c,t}$. The negative sign in front of the transitory components refers to the fact that when expected growth (inflation) increases, the nominal short-term rate rises as well and the yield curve spread shrinks.

In contrast to the traditional carry strategy, sorting countries according to their relative yield curve's slope produces relevant reallocations across portfolios over time. In order to replicate this empirical fact, we calibrate our model so that the country-specific fixed effects are nearly irrelevant, that is, we have $\overline{slope_i}^\infty \approx \overline{slope_j}^\infty \quad \forall i,j$. Since unconditional level of the yield curve slope is increasing in both β_π and β_c, countries featuring high (low) β_π and high (low) β_c tend to have similar unconditional slopes. We anticipate that this combination of sensitivity coefficients applies to both the data and our calibration.

Hence in our model the placement of countries in different slope-sorted portfolios depends mainly on $\beta_\pi^i x_{\pi,t} + \beta_c^i x_{c,t}$ for $i = 1, ..., 10$. We analyse the time behavior of this processes by means of simulations in section 4.

Slope carry excess returns. The expected excess return of a strategy that is short the risk-free rate of base country b and long the infinite horizon bond of country i for one period is

$$\begin{aligned} \log E_t \left[RX^{\infty}_{i,t+1}\right] &= \log E_t \exp\left\{-r_{b,t} + hpr^{\infty}_{i,t+1} + \Delta e_{bi,t+1}\right\} \\ &= V_t \left[m_{b,t+1}\right] - cov_t \left(m^P_{i,t+1}, m_{b,t+1}\right). \end{aligned} \quad (12)$$

where the last equality follows from equation (7), that is, from the observation that the exchange rate perfectly hedges $hpr^{\infty}_{i,t+1}$ in our complete markets economy. After normalizing the coefficients of the base country so that $\beta^b_c = \beta^b_\pi = 1$, we get:

$$\log E_t \left[RX^{\infty}_{i,t+1}\right] = \log E_t \left[RX^{1}_{i,t+1}\right] - \beta^c_i \left[\frac{k_{\varepsilon c}\sigma^2_{xc}}{1-\rho_c} - \frac{\rho_{c\pi}k_{\varepsilon\pi}\sigma^2_{x\pi}}{(1-\rho_c)(1-\rho_\pi)}\right] + \beta^\pi_i \frac{k_{\varepsilon\pi}\sigma^2_{x\pi}}{1-\rho_\pi}. \quad (13)$$

Equation (13) shows three important results. First, this strategy exposes the investor to the same extent of currency risk that we have seen for the traditional carry ($\log E_t \left[RX^{1}_{i,t+1}\right]$). Second, the investor is also exposed to the risk associated with the holding period return of the long-maturity bond. Specifically, when good news for long-run growth materialize, either directly ($\varepsilon_{c,t+1} > 0$) or indirectly ($\varepsilon_{\pi,t+1} < 0$ and assuming $\rho_{c\pi} < 0$), yields increase and the infinite-maturity bond produces a loss in states of world with low marginal utility. As a result, this strategy provides a hedge against global growth news shocks, thus commanding a negative risk premium (see middle term in equation (13)).

Third, this strategy commands a positive risk premium with respect to expected global inflation news (last term in equation (13)). Nominal yields increase when positive news to expected inflation materialize, thus resulting in a negative holding period return in high-marginal utility states. This inflation risk premium is increasing in

β_π^i. Furthermore, we anticipate that under our benchmark calibration, the last term in equation (13) accounts for about 55% of the excess return $\log E_t\left[RX_{i,t+1}^{\infty}\right]$. Equivalently, the inflation risk premium is the key driver of the excess return on foreign long-term bonds investments, and investing in the long-term bonds of high β_i^π countries should command a premium over investing in the long-term bonds of low β_i^π countries.

In the next section, we calibrate the model and assess its quantitative performance. When doing so, we consider a cross section of β_c^i and β_π^i consistent with our empirical estimates, and analyze the currency returns through simulations that reflect the estimated dynamics of expected growth and inflation.

4 Calibration and Simulations

We detail our baseline quarterly calibration in Table 5. The subjective discount factor δ is set to reflect an average annualized nominal risk free rate of 4.2%, consistent with the data. The risk aversion parameter is equal to 10. This value enables us to match the conditional expected value of the returns from the slope carry strategy. The parameters $\overline{\mu}_\pi$ and $\overline{\mu}_c$ are chosen to reflect an average annual growth rate of 2.6%, and an average inflation rate of 1%, as in the data.

Global expected consumption growth (x_c) and inflation (x_π) are modeled according to equation (3). We calibrate the autocorrelation parameters ρ_c and ρ_π to be consistent with our estimates of equation (3). The consumption-inflation feedback parameter $\rho_{c\pi}$ is set equal to -0.05, again consistent with our estimation. The volatility of our short-run consumption shocks, σ_c, and that of our long-run news shocks about global consumption growth, σ_{xc}, are chosen to target the average volatility and autocorrelation of consumption growth in our data set, respectively. We apply a similar strategy

TABLE 5: Calibration

Description	**Parameter**	**Value**	**Estimate/ Moment**
Subjective discount factor	δ	0.997	Avg.$_i$ $\left[E(r^f)\right]$
Risk Aversion	γ	10	$E(carry^S)$
Cross-country average consumption growth	$\bar{\mu}_c$	0.49%	0.54%
			(0.05%)
Volatility of cons growth short-run shock	σ_c	0.46%	Avg.$_i$ $[\sigma(\Delta c)]$
Volatility of cons growth long-run shock	σ_{xc}	0.11%	Avg.$_i$ $[ACF_1(\Delta c)]$
Autocorr. cons growth long-run risk	ρ_c	0.810	0.570
			(0.110)
Cross-country average inflation growth	$\bar{\mu}_\pi$	0.25%	0.41%
			(0.05%)
Volatility of inflation short-run shock	σ_π	0.55%	Avg.$_i$ $[\sigma(\pi)]$
Volatility of inflation long-run shock	$\sigma_{x\pi}$	0.11%	Avg.$_i$ $[ACF_1(\pi)]$
Aucocorr inflation long-run risk	ρ_π	0.988	0.910
			(0.040)
Cons growth / inflation long-run feedback	$\rho_{c\pi}$	-0.050	-0.050
			(0.030)

Notes - This table reports the value of our parameters under our baseline calibration. Some parameters are calibrated to be within the confidence intervals of their counterpart estimated in the data. HAC-corrected standard errors are reported in the parentheses. Other parameters are calibrated to match cross sectional averages (Avg.$_i$) in the data. Empirical estimates are from the specification detailed in equation (3). Our data set is detailed in section 2.

for the volatility parameters in the inflation process.

We generate cross-sectional differences across countries by setting heterogeneous exposure to both global consumption, β_c^i, and inflation, β_π^i, for ten different countries. Our detailed calibration of these parameters is reported in the appendix (table E.1) and is broadly consistent with our estimates described in section 2. Given our cross section of exposure parameters, we generate country-level mean growth and inflation by spreading around the mean values μ_c and μ_π according to the parsimonious formula in equations (4) and (6). Given these parameters, we simulate the model for 100 quarters and show average results across 1,000 simulations.

TABLE 6: Heterogeneous Exposure and Cross-sectional Moments

	Avg.$_i$			StDev$_i$/Avg.$_i$		
	Value	***(Std Err)***	**Model**	**Value**	***(Std Err)***	**Model**
β_c	0.77	(0.06)	0.64	0.57	(0.13)	0.33
$E(\Delta c)$	2.22	(0.14)	2.64	0.34	(0.04)	0.15
$\sigma(\Delta c)$	1.05	(0.09)	0.97	0.26	(0.05)	0.04
$ACF_1(\Delta c)$	0.24	(0.09)	0.42	0.17	(0.04)	0.06
β_π	1.00	(0.14)	0.99	0.51	(0.18)	0.39
$E(\pi)$	1.70	(0.15)	0.99	0.41	(0.06)	0.39
$\sigma(\pi)$	1.02	(0.15)	1.84	0.22	(0.04)	0.24
$ACF_1(\pi)$	0.14	(0.11)	0.53	0.18	(0.04)	0.35
$Corr(\Delta c, \pi)$	-0.22	(0.10)	-0.11	-0.26	(0.05)	-0.25

Notes - The table reports cross sectional averages (**Avg.**$_i$) and cross sectional coefficients of variation (**StDev**$_i$/**Avg.**$_i$) for several moments of interest. The column 'Value' reports our point estimates computed using the data set described in Section 2. We report the associated HAC-adjusted standard errors under the column 'Std Err'. The entries for the column 'Model' are obtained by simulating 1,000 short samples comprising 100 quarterly observations. Simulated data are time aggregated at the annual frequency. All parameters are set to their benchmark values reported in Table 5. For $ACF_1(\Delta c)$ and $ACF_1(\pi)$, we report **StDev**$_i$ rather than **StDev**$_i$/**Avg.**$_i$.

In table 6, we focus on key moments of both consumption and inflation for our cross section of countries. For the sake of parsimony, we report global averages and cross-sectional dispersion. Specifically, we report cross-sectional averages, label as "Avg.$_i$", of moments simulated in the time-series at the country level. In order to measure heterogeneity across countries we also report the cross-sectional coefficient of variation, labeled as "StDev$_i$/Avg.$_i$", of these moments across our ten countries.

Our model fits well the data as our simulated moments are in line with our empirical confidence intervals. We point out two minor limitations. First, our inflation processes are on average slightly more volatile and persistent than in the data. Second, we produce a cross-sectional variation in both the average and the volatility of consumption growth that is slightly smaller than in the data. This issue could be easily resolved by (i) introducing country-specific volatility for short-run consumption growth shocks; and (ii) enriching the link between average consumption growth and exposure to growth news shocks stated in equation (4). Since these variations would improve our results at the cost of tractability, we decided to abstract away from them and focus on our constrained– and hence more conservative– calibration.

4.1 Simulating Expectations

To preserve tractability, we focus on a setting with log-normally distributed shocks. Within this setting, we model variations in expected inflation and economic growth as the realization of a large joint negative news shock.

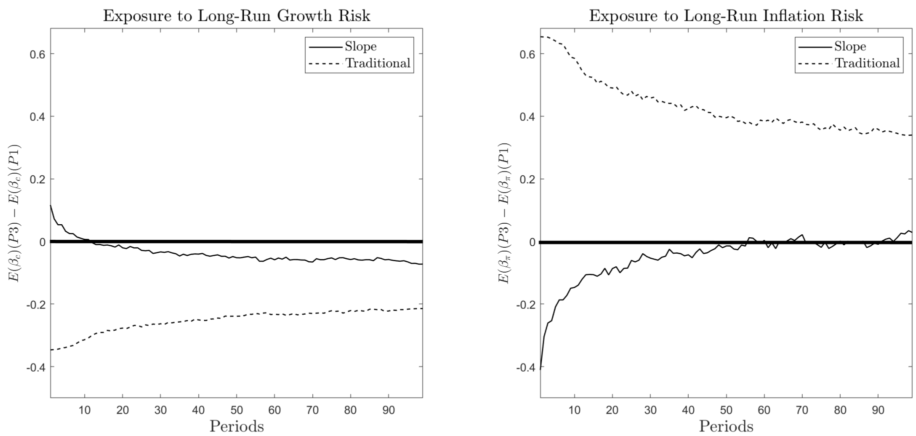
Specifically, we think of the *pre-break* period as a sub-sample in which both of our state variables, $x_{c,t}$ and $x_{\pi,t}$, start from positive values. Consistent with our empirical evidence, we set the initial point of expected global growth and inflation so that $x_{c,0} = x_{\pi,0} = 0.125\%$, i.e., both processes capture above-average expectations. At the time of the break, $t = t^*$, our agents receive negative news shocks about both expected growth and inflation, so that in the *post-break* sample expectations decline below average: $x_{c,t^*} = x_{\pi,t^*} = -0.5\%$. We simulate 1000 different samples with 100 quarterly observations, and introduce this low-probability event at $t^* = 51$, consistent with our empirical pre-break sample.

Within each sub-sample, we are interested in sorting countries according to either their short-term rate or the slope of their yield curve, at each point in time. The first characteristic is key in forming portfolios used in the traditional carry strategy. The yield curve slope, on the other hand, is important in forming portfolios for the slope carry. This simulation exercise is relevant for at least two reasons. First, it accounts for the endogenous probability of a country to be reallocated across portfolios depending on the chosen sorting variable, that is, either the level or the slope of the yield curve. Second, it enables us to compute time-varying properties at the portfolio level.

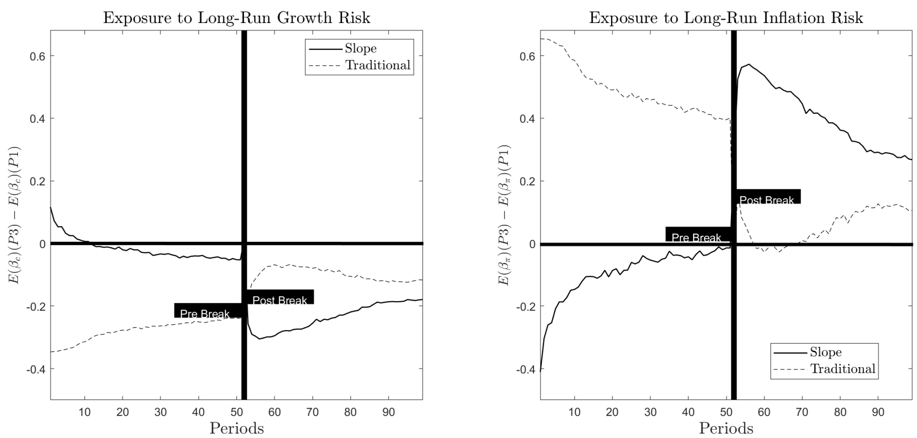
To illustrate the extent of this time variation, in figure 3, we report the GDP-weighted exposure coefficients across portfolios both with respect to growth and inflation news shocks (i.e. the weighted averages of β_c and β_π for each portfolio). In the top panels, we depict the case in which there is no extreme variation, whereas the bottom panels include a sizeable negative shock. In both cases, expectations are initialized to be above their unconditional levels. In the next subsections, we describe in detail how this time variation is relevant for the traditional and slope carries.

Traditional and slope carry without a break. In figure 3(a), we depict the behavior of the exposure coefficients of our carry strategies in the scenario in which there is no break. In our model, the traditional carry strategy features a negative exposure to global growth news shocks, as the investor in the base country borrows in high-β_c currencies and invests in low-β_c currencies. When we initialize our pre-break subsample, the top-three (bottom-three) β_c countries end up in the low (high) risk-free rate portfolio, henceforth "P1" ("P3"). Because of (i) mean reversion, and (ii) the fact that our country fixed effects in the nominal risk-free rates are moderate (see equation (9) and table E.2 in the appendix), the exposure of the traditional carry tends to decrease in absolute value as some of the countries with intermediate levels of β_c enter more frequently in portfolios P1 and P3 due to fluctuations in $\beta_c^i x_{c,t} + \beta_\pi^i x_{\pi,t}$. A similar logic applies to the exposure of the traditional carry to inflation risk, meaning that it is very positive at the beginning of our simulation and it decreases in magnitude over time.

Turning to the slope carry, we point out that in the middle of our sample it has a slightly negative exposure to global growth risk and a nearly null exposure to inflation risk. As a result, this strategy should bear an unconditional risk premium close to zero.



(a) Without Break



(b) With Break

FIG. 3 - Simulated Portfolio-Level Exposures. This figure shows differences in simulated portfolio-level exposures to long-run growth risk (β_c, left panels) and expected global inflation news shocks (β_π, right panels). In panel (a), there is no break in expected growth and inflation. In panel (b), a break suddenly reduce both expected growth and inflation. For the traditional carry, portfolio P1 (P3) comprises low-interest rate (high-interest rate) countries. For the slope carry, portfolio P1 (P3) comprises flatter-yield curve (steeper-yield curve) countries. Our quarterly calibration is detailed in table 5. At the break point, both expected global growth and expected global inflation decline as in the data (see figure 2). We depict averages across repetitions of small sample in which both expected inflation and growth are initialized above their unconditional average.

Traditional and slope carry with a break. In figure 3(b), we depict the behavior of the exposure coefficients of our carry strategies in the scenario in which there is a break, i.e., a substantial downward revision in expectations. Qualitatively, the behavior of the traditional carry exposures remain the same. In the post-break sample, the magnitude of the exposure coefficients is reduced compared to the no-break scenario, but their signs are unchanged.

In contrast, the break changes substantially the exposures of the slope carry both in terms of magnitude and in terms of sign. In the aftermath of a joint negative shock to the growth and inflation expectations, steeper-slope countries feature higher β_π and lower β_c. In order to understand this dramatic change of sign, we note that the unconditional slopes are very similar across countries (see table E.2 in the appendix), implying that the slope-based ranking of our countries is almost entirely driven by the transitory components $\beta_c^i x_{c,t} + \beta_\pi^i x_{\pi,t}$.

More specifically, in our model dispersion in β_π's is more pronounced than that in β_c's. As a result, the relative slopes of the yield curves are mainly driven by exposure to inflation, consistently with our empirical findings reported in table B.2. Hence, in the context of our simulations, we can note that $slope_{i,t}^\infty - slope_{j,t}^\infty \approx -(\beta_\pi^i - \beta_\pi^j) x_{\pi,t}$. If we consider the situation in which country i has higher inflation exposure than country j, then $\textbf{sign}(\beta_\pi^i - \beta_\pi^j) = 1$, and

$$\textbf{sign}\left(slope_{i,t}^\infty - slope_{j,t}^\infty\right) = -\textbf{sign}\left(x_{\pi,t}\right). \tag{14}$$

Equivalently, when expected global inflation is below average, the yield curves of high-β_π countries tend to be steeper, whereas the opposite is true when expected inflation is above average. Given the decline in both expected global inflation and long-run growth that we have estimated in the data post-break, we can think of the slope carry as going long (short) in high-β_π countries post-break (pre-break). Since in our data-driven calibration there is a mild negative correlation between β_c's and β_π's, the slope carry also features a negative exposure to growth news shocks post-break.

In the next section, we analyze how these endogenous time-varying exposures of our portfolios affect currency risk premia in equilibrium.

4.2 Impulse response functions and risk premia

Impulse responses. In figure 4, we show the response of our portfolios to adverse shocks to expected global growth and inflation. In both cases, the marginal utility of the investor in the base country increases, meaning that we are looking at high marginal utility states.

Consistent with our analysis of the portfolio exposures, we see that the traditional carry has a negative exposure to growth news shocks both before and after the break. As a result, this strategy must pay a positive risk premium against long-run global growth risk. We note also that given our calibration, this strategy has a negative exposure to global inflation shocks, that is, its holding period return is negative in high-marginal utility states and hence it must pay a positive risk premium also with respect to inflation shocks.

The behavior of the slope carry returns deserves more attention. Pre-break, this strategy produces positive excess returns with respect to both negative growth news shocks and positive inflation news shocks. Hence this strategy provides insurance against both sources of global risk. In the post-break period, however, the opposite holds. Furthermore, we note that the responses to inflation shocks are much more pronounced compared to those relative to growth news shocks. Equivalently, the contribution of the risk premium of inflation risk appears to dominate in our simulations.

Given these observations, let us focus solely on the role of inflation risk in what follows. Recall from equation (14) that when $x_{\pi,t} > 0$ ($x_{\pi,t} < 0$), a steeper-slope (henceforth S) country features low-β_π (high-β_π). The opposite is true for the flatter-slope

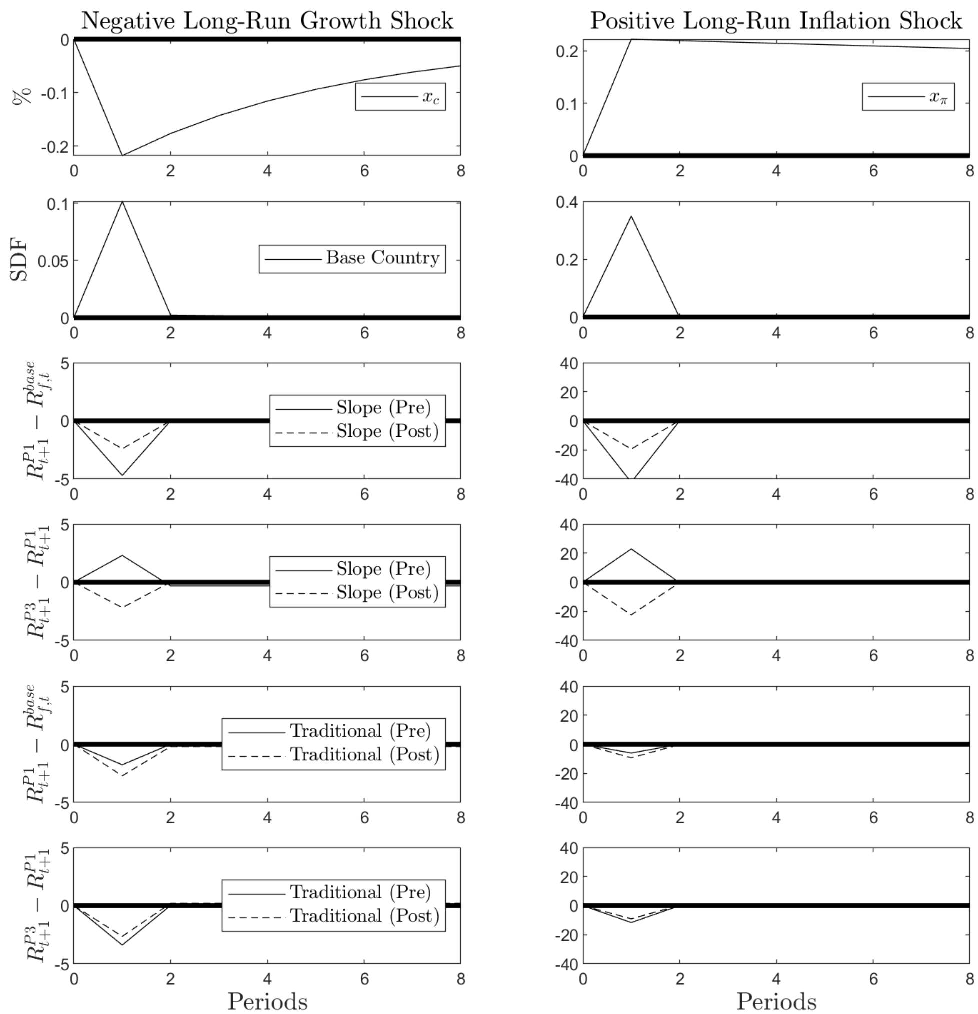


FIG. 4 - Portfolios Response to Global News Shocks. This figure shows portfolio-level impulse response functions for the Pre-Break (solid line) and Post-Break (dashed line) period. The left (right) panels report the response to adverse global consumption growth (inflation) news shocks. SDF refers to the stochastic discount factor in the base country. $R^{P1}_{t+1} - R^{base}_{f,t}$ is the excess return that an investor in the base country obtains by investing in the low-portfolio, P1. $R^{P3}_{t+1} - R^{P1}_{t+1}$ refers to the excess returns of the carry strategy. For the traditional (slope) carry, P1 comprises low-interest rate (flat-yield curve) countries. Our quarterly calibration is detailed in table 5.

(henceforth F) country. Under these conditions, the conditional risk premium of the slope carry can be computed as:

$$
\begin{aligned}
E[carry^S|x_{\pi,t}] &:= \log E_t\left[RX^{\infty}_{\mathbf{S},t+1}\right] - \log E_t\left[RX^{\infty}_{\mathbf{F},t+1}\right] \\
&\approx -\mathbf{sign}(x_{\pi,t})\underbrace{(\beta^H_\pi - \beta^L_\pi)}_{>0}\underbrace{\frac{k_{\varepsilon\pi}\sigma^2_{x\pi}}{1-\rho_\pi}}_{>0},
\end{aligned}
\tag{15}
$$

where the second row of (15) is an approximation about $\beta^H_c = \beta^L_c$ or, equivalently, the risk premium obtained by abstracting away from the role of growth news shocks.

Equation (15) confirms three relevant points. First, the slope carry strategy features endogenously time-varying exposure to news shocks because the countries that end up in the two legs of the strategy change with the expectations (i.e., $x_{\pi,t}$). Second, the slope carry should produce a positive risk premium in periods in which expected global inflation is below average, consistent with our empirical evidence. Third, its unconditional risk premium should be zero since $E[\mathbf{sign}\,(x_{\pi,t})] = 0$.

Simulated moments. One key advantage of the tractability of our model is that it features an exact solution and hence it can be simulated without approximation errors. We report key equilibrium moments in table 7.

Our model captures the key results that we have highlighted in our empirical investigation. Specifically, it produces a nearly null slope carry risk premium in our full sample while simultaneously matching the magnitude of its positive risk premium post-break. Turning our attention to the mid- and bottom-part of the table, we see that these quantitative results have been obtained with a dispersion of the slope across our simulated portfolios that is very consistent with that observed in the data. The same is true for our simulated exposures both in the full sample and in the post-break sample. These observations are relevant because our model replicates almost entirely the observed slope carry while simultaneously reproducing plausible

TABLE 7: Simulated Moments

	Traditional Carry		Slope Carry	
	Data	Model	Data	Model
$E(carry)$ (Full Sample)	4.93	2.35	2.62	2.05
	(2.09)		(2.24)	
$E(carry)$ (Post-08/07)	0.79	0.88	6.17	7.36
	(3.43)		(3.32)	
$E($*sorting var*$)$ P3 - $E($*sorting var*$)$ P1	4.02	1.85	1.52	1.73
	(0.07)		(0.03)	
$E($*sorting var*$)$ P3 - $E($*sorting var*$)$ P1 (Post-08/08)	2.93	1.47	1.24	1.94
	(0.07)		(0.04)	
$E(\Delta FX)$ P3 - $E(\Delta FX)$ P1	0.76	0.53	0.85	2.21
	(0.61)		(0.59)	
$E(\Delta FX)$ P3 - $E(\Delta FX)$ P1 (Post-08/08)	-2.66	-0.53	3.49	3.80
	(1.02)		(0.99)	
$E(\beta_c)$ P3 - $E(\beta_c)$ P1	-0.68	-0.16	0.14	-0.06
$E(\beta_c)$ P3 - $E(\beta_c)$ P1 (Post-08/08)	-0.64	-0.05	-0.26	-0.20
$E(\beta_\pi)$ P3 - $E(\beta_\pi)$ P1	-0.09	0.30	0.04	0.15
$E(\beta_\pi)$ P3 - $E(\beta_\pi)$ P1 (Post-08/08)	-0.35	-0.04	0.27	0.55

Notes - This table reports both empirical and simulated moments for both the traditional and the slope carry strategies. All moments are (i) computed as GDP-weighted averages within each portfolio, (ii) annualized, and (iii) multiplied by 100 (except for β_c and β_π). For the traditional (slope) carry, the high-portfolio, P3, comprises countries with high short-term interest rate (steeper yield curve slope). The opposite is true for the low-portfolio, P1. The entries for the moments are based on 1,000 simulations of 100 quarters. All parameters are set to their benchmark values reported in Table 5. $E(\Delta FX)$ refers to the average exchange rate depreciation. $E(\beta_c)$ ($E(\beta_\pi)$) measures the portfolio-level exposure to global expected consumption growth (inflation). The numbers in parenthesis denote standard errors.

cross-sectional spreads for both yield curve slopes and exposure coefficients.

In addition, our model captures the increasing contribution of the exchange rate to the slope carry. In the post-break sample, the contribution of the exchange rate to the slope carry has been 3.49% in the data. The model produces a similar value, in the order of 3.80%. Hence our model captures an interesting dimension of the composition of the slope carry and it does not rely solely on the bonds holding period return.

Finally, we note that similar considerations apply to the traditional carry. Hence our model matches (i) key conditional properties of the slope carry through heterogeneous exposure to global inflation news shocks, and (ii) key unconditional properties of the traditional carry through both the inflation and the growth news shock channel.

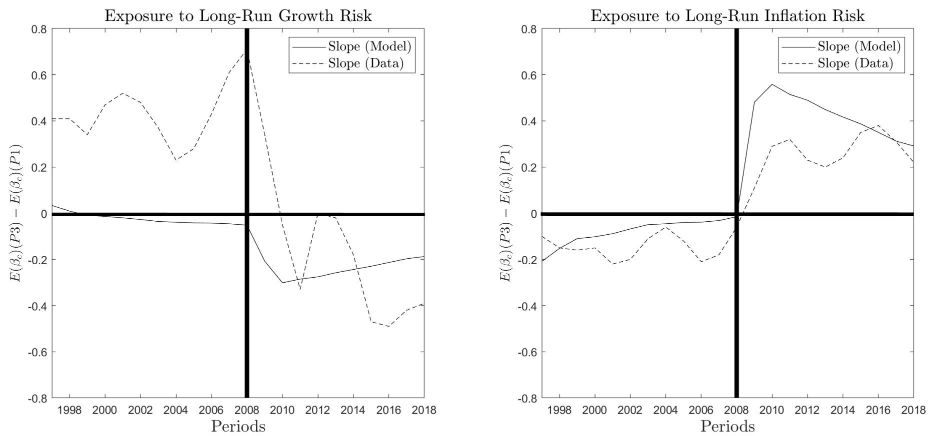


FIG. 5 - Slope Carry: Portfolio-Level Exposures for Model and Data. This figure shows differences in portfolio-level exposures to long-run growth risk (β_c, left panel) and expected global inflation news shocks (β_π, right panel) for the slope carry. The solid lines are obtained using simulated data. Our quarterly calibration is detailed in table 5. The dashed lines show the empirical estimates obtained using the data set of section 2. Portfolio P1 (P3) comprises flatter-yield curve (steeper-yield curve) countries. At the break point (August 2007) both expected global growth and expected global inflation decline as in the data (see figure 2). For the simulated data, we depict averages across repetitions of small sample in which both expected inflation and growth are initialized above their unconditional average.

Additional empirical validation. We conclude this section by looking at dynamics that we did not directly target in our calibration and that we consider as an external validation of our model. Specifically, in figure 5, we compare the slope carry exposure to expectations about global growth and inflation in both our data and our model.

Our model captures the decline in the exposure of the slope carry strategy to growth news shocks. Even though this phenomenon is less pronounced than in the data, this result is more than satisfactory since growth news shocks have a limited quantitative contribution to the conditional average slope carry premium both in the data and in the model. Turning our attention to the exposure to global inflation news shocks, in contrast, we see that the model conforms very well with the dynamics of our empirical measure. We consider this result as very supportive of both our calibration and our quantitative model.

5 Extended Model

In this section, we extend our model in two dimensions. First, we explore the role of the intertemporal elasticity of substitution (IES) by considering Epstein and Zin (1989a) preferences. Next, we introduce a demand shock and explore the relevance of global inflation shocks for the volatility of domestic yields. All of our derivations are reported in appendix F and follow the same steps of those reported for the special case with EIS=1.

5.1 The role of the IES

We replicate our analysis by adopting the following preferences,

$$U_{i,t} = \left\{ (1-\delta) C_{i,t}^{1-\frac{1}{\psi}} + \delta E_t \left[U_{i,t+1}^{1-\gamma} \right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\psi}}},$$

where ψ and γ determine the IES and the relative risk aversion, respectively. These preferences imply the following real stochastic discount factor,

$$m_{i,t+1}^{\text{real}} = \theta \log \delta - \frac{\theta}{\psi} \Delta c_{i,t+1} + (\theta - 1) r_{i,t+1}^{c},$$

where $r_{i,t}^{c}$ is the return on the consumption claim and $\theta := \frac{1-\gamma}{1-\frac{1}{\psi}}$. In each country, the nominal discount rate is still determined as $m_{i,t+1} = m_{i,t+1}^{real} - \pi_{i,t+1}$.

Up to a log-linearization, our extended model (i) preserves the affine structure of our benchmark setting, and (ii) differs from our baseline case because the IES is no longer forced to be equal to one. In table 8, we compare our simulated results when we set $IES = 2$, a typical number in the international macro-finance literature (see, for example, Colacito and Croce 2011b). Keeping everything else constant, a higher

TABLE 8: The Role of IES

	Traditional Carry			Slope Carry		
	Data	**IES=1**	**IES=2**	**Data**	**IES=1**	**IES=2**
$E(carry)$ (Full Sample)	4.93 (2.09)	2.35	1.63	2.62 (2.24)	2.05	4.12
$E(carry)$ (Post-08/07)	0.79 (3.43)	0.88	0.35	6.17 (3.32)	7.36	9.61
E(*sorting var*) P3 - E(*sorting var*) P1	4.02 (0.07)	1.85	1.75	1.52 (0.03)	1.73	1.80
E(*sorting var*) P3 - E(*sorting var*) P1 (Post-08/08)	2.93 (0.07)	1.47	1.39	1.24 (0.04)	1.94	2.18
$E(\Delta FX)$ P3 - $E(\Delta FX)$ P1	0.76 (0.61)	0.53	-0.10	0.85 (0.59)	2.21	2.15
$E(\Delta FX)$ P3 - $E(\Delta FX)$ P1 (Post-08/08)	-2.66 (1.02)	-0.53	-1.02	3.49 (0.99)	3.80	3.44
$E(\beta_c)$ P3 - $E(\beta_c)$ P1	-0.68	-0.16	-0.14	0.14	-0.06	-0.09
$E(\beta_c)$ P3 - $E(\beta_c)$ P1 (Post-08/08)	-0.64	-0.05	-0.02	-0.26	-0.20	-0.22
$E(\beta_\pi)$ P3 - $E(\beta_\pi)$ P1	-0.09	0.30	0.27	0.04	0.15	0.31
$E(\beta_\pi)$ P3 - $E(\beta_\pi)$ P1 (Post-08/08)	-0.35	-0.04	-0.10	0.27	0.55	0.64

Notes - This table reports both empirical and simulated moments for both the traditional and the slope carry strategies. All moments are (i) computed as GDP-weighted averages within each portfolio, (ii) annualized, and (iii) multiplied by 100 (except for β_c and β_π). For the traditional (slope) carry, the high-portfolio, P3, comprises countries with high short-term interest rate (steeper yield curve slope). The opposite is true for the low-portfolio, P1. The entries for the moments are based on 1,000 simulations of 100 quarters. All parameters are set to their benchmark values reported in Table 5, and the IES is also allowed to be 2. $E(\Delta FX)$ refers to the average exchange rate depreciation. $E(\beta_c)$ ($E(\beta_\pi)$) measures the portfolio-level exposure to global expected consumption growth (inflation). The numbers in parenthesis denote standard errors.

IES reduces the spread in the interest rates and hence it reduces the profitability of the traditional carry both over the full sample and in the post-2008 period.

In contrast, the slope carry increases (decreases) post-2008 (pre-2008). This is because the SDF features loadings with respect to both growth and inflation news that are larger than before, as they depend on $\gamma - 1/\psi > \gamma - 1$ when $\psi > 1$. Equivalently, going back to equation (11), the coefficients $k_{\epsilon c}$ and $k_{\epsilon \pi}$ are more sizable.

Looking at all of the other moments, we find only marginal variations in our simulated models when we increase our IES from one to two.

5.2 Determinants of treasury yields: the role of demand shocks

Duffee (2018) documented that inflation expectation shocks explain a relatively small fraction of the variability of Treasury yields in the US. We show that introducing demand shocks can easily preserve our main results and enable our setting to be consistent with this empirical finding.

Specifically, we assess the role of global inflation shocks in an extended version of our model with (i) IES=2, and (ii) a common demand shifter, i.e., we introduce demand shocks that affect all countries (Albuquerque *et al.* 2016). In this section, we are agnostic about the exposure of each country to global demand shocks and we set it to be identical across countries.[1] Given this assumption, global demand shocks do not alter currency risk premia, because they affect all countries to the same extent. Equivalently, demand shocks affect the variance of local yields without affecting their cross sectional properties.

We enrich our preferences by introducing a process, Λ_t, that functions as a demand shifter:

$$U_{i,t} = \left\{ (1-\delta)\Lambda_t C_{i,t}^{1-\frac{1}{\psi}} + \delta E_t \left[U_{i,t+1}^{1-\gamma} \right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\psi}}}.$$

These preferences imply the following real stochastic discount factor,

$$m_{i,t+1}^{\text{real}} = \theta \log \delta + \theta \Delta\lambda_{t+1} - \frac{\theta}{\psi}\Delta c_{i,t+1} + (\theta - 1) r_{i,t+1}^{c}$$

where $\Delta\lambda_{t+1}$ evolves as follows:

$$\Delta\lambda_{t+1} := \log(\Lambda_{t+1}/\Lambda_t) = x_{d,t},$$

[1]An analysis of heterogeneous exposure to demand shocks is left for future research.

TABLE 9: The Role of Demand Shocks

Panel A: International Moments

	Traditional Carry			Slope Carry		
Demand shock	**yes**	**yes**	**no**	**yes**	**yes**	**no**
Demand shock downward jump	**yes**	**no**	**–**	**yes**	**no**	**–**
$E(carry)$ (Full Sample)	1.17	1.17	1.63	1.08	1.08	4.12
$E(carry)$ (Post-08/07)	0.09	0.09	0.35	5.63	5.63	9.61
$E(sorting\ var)$ P3 - $E(sorting\ var)$ P1	1.69	1.69	1.75	1.65	1.65	1.80
$E(sorting\ var)$ P3 - $E(sorting\ var)$ P1 (Post-08/08)	1.39	1.39	1.39	1.82	1.82	2.18
$E(\Delta FX)$ P3 - $E(\Delta FX)$ P1	-0.50	-0.50	-0.10	1.66	1.66	2.15
$E(\Delta FX)$ P3 - $E(\Delta FX)$ P1 (Post-08/08)	-1.27	-1.27	-1.02	2.70	2.70	3.44

Panel B: Local Moments

Share of volatility due to inflation	hpr^{∞}	$Slope$	ΔFX
With demand shock	9.5%	60%	25%
Without demand shock	79%	82%	28%

Notes - This table reports both empirical and simulated moments for both the traditional and the slope carry strategies. All moments are (i) computed as GDP-weighted averages within each portfolio, (ii) annualized, and (iii) multiplied by 100 (except for β_c and β_π). For the traditional (slope) carry, the high-portfolio, P3, comprises countries with high short-term interest rate (steeper yield curve slope). The opposite is true for the low-portfolio, P1. The entries for the moments are based on 1,000 simulations of 100 quarters. All parameters are set to their benchmark values reported in Table 5, except the IES that is set to 2. When the demand shock is present, we set $\sigma_d = 5e^{-4}$ and $\rho_d = .9742$. When we include a downward jump in the demand process, we set it equal to -1StDev(x_d). $E(\Delta FX)$ refers to the average exchange rate depreciation. $Slope$ and hpr^{∞} refer to the slope and the holding period return of an infinite-maturity bond, respectively. The share of volatility refers to the simple average of the country-level shares. The numbers in parenthesis denote standard errors.

and

$$x_{d,t} = \rho_d x_{d,t-1} + \sigma_{x,d}\varepsilon_{d,t}.$$

We assume that the innovation to the demand shifter are *i.i.d.N(0,1)* and simulate our model under two different scenarios. First, we assume that no additional demand shock takes place at $t^* = 2008$. Under the second scenario, instead, we assume that at the time of the break, i.e., $t = t^*$, our agents receive also a negative demand shocks, similarly to what we did with expected global inflation and growth. Across both scenarios, we set $\sigma_d = 5e^{-4}$ and $\rho_d = .9742$, two values that are conservative with respect to Albuquerque *et al.* (2016).

We report our simulation results in table 9. First of all, we note that our main results are preserved when we introduce demand shocks. The traditional carry declines by about 50 basis points, whereas our slope carry decreases by 400 basis points. Both moments, however, remain empirically plausible. The risk-free rates, the slopes and the exchange rate depreciation rates show no significative change across portfolios. Second, we point out the inclusion of a global drop in demand is immaterial for our analysis.

Furthermore, turning our attention to panel B of table 9, we see that including demand shocks enables us to reduce significantly the share of volatility of local yields explained by inflation shocks. This result confirms that global news shocks about inflation can be a key determinant of international carry strategy even though they explain a small portion of the dynamics of local yield curves.

6 Conclusion

In this paper, we provide novel empirical evidence regarding the performance of carry trade strategies based on sorting the cross section of currencies on the level and on the slope of their yield curves. In particular, we revisit the conclusion of the extant literature concerning the near zero average excess return associated to being long in steeper yield curve countries and short in flatter yield curve countries (slope carry). We note that the risk premium on this strategy is very negative before 2008 and it turns sharply positive in more recent years. Equivalently, the null excess return over a long sample conceals the profitability of the slope carry over different sub-samples.

We explain these empirical findings by augmenting an otherwise standard international asset pricing model with two sources of empirically motivated cross-country heterogeneity. Namely, we focus on heterogeneous exposure to news shocks about both expected global consumption growth and inflation. We document that in our equilibrium model, heterogeneity about expected economic growth explains the performance of portfolios sorted on the level of the yield curve (traditional carry), whereas heterogeneity with respect to inflation is key to account for the average returns of the slope carry within different sub-samples.

Future developments should extend this setting to international real business cycle models to study the role of international investment flows and international frictions for the cross section of currency risk premia. They should also analyze the role of the zero lower bound (see, among others, Caballero *et al.* (2016)) on the profitability of currency strategies in the aftermath on the Global Financial Crisis.

References

Albuquerque, R., M. Eichenbaum, V. X. Luo, and S. Rebelo, (2016), Valuation risk and asset pricing, *The Journal of Finance* 71, 2861–2904.

Alvarez, F. and U. Jermann, (2005), Using asset prices to measure the persistence of the marginal utility of wealth, *Econometrica* 73, 1977–2016.

Avdjiev, S., B. Hardy, S. Kalemi-Ozcan, and S. L, (2020), Gross capital flows by banks, corporates andsovereigns, *Working Paper* .

Bakshi, G., M. Cerrato, and J. Crosby, (2017), Implications of incomplete markets for international economies, *Review of Financial Studies* .

Bansal, R. and I. Shaliastovich, (2013), A long-run risks explanation of predictability puzzles in bond and currency markets, *Review of Financial Studies* 26, 1–33.

Barro, R., (2006), Rare disasters and asset markets in the twentieth century, *Quarterly Journal of Economics* 121, 823–866.

Beeler, J. and J. Y. Campbell, (2012), The long-run risks model and aggregate asset prices: An empirical assessment, *Critical Finance Review* 1, 141–182.

Caballero, R., E. Farhi, and P.-O. Gourinchas, (2016), Global imbalances and currency wars at the zlb, *Working Paper* .

Caballero, R. J., E. Farhi, and P.-O. Gourinchas, (2008), An equilibrium model of ”global imbalances” and low interest rates, *American Economic Review* 98, 358–393.

Chabi-Yo, F. and R. Colacito, (2019), The term structures of coentropy in international financial markets, *Management Science* 65, 3541–3558.

Chernov, M., J. Graveline, and I. Zviadadze, (2018), Crash risk in currency returns, *Journal of Fiancial and Quantitative Analysis* .

Colacito, R., M. M. Coce, F. Gavazzoni, and R. Ready, (2018), Currency risk factors in a recursive multicountry economy, *The Journal of Finance* 73, 2719–2756.

Colacito, R. and M. M. Croce, (2011a), Risks for the long run and the real exchange rate, *Journal of Political Economy* 119, 153–81.

Colacito, R. and M. M. Croce, (2011b), Risks for the long run and the real exchange rate, *Journal of Political Economy* 119, 153–182.

Coppola, A., M. Maggiori, B. Neiman, and J. Schreger, (2020), Redrawing the map of global capital flows: The role of cross-border financing and tax havens, Working Paper 26855, National Bureau of Economic Research.

Della Corte, P., T. Ramadorai, and L. Sarno, (2016a), Volatility risk premia and exchange rate predictability, *Journal of Financial Economics* 120, 21 – 40.

Della Corte, P., S. J. Riddiough, and L. Sarno, (2016b), Currency Premia and Global Imbalances, *The Review of Financial Studies* 29.

Della Corte, P., L. Sarno, and I. Tsiakas, (2009), An Economic Evaluation of Empirical Exchange Rate Models, *Review of Financial Studies* 22, 2491–530.

Della Corte, P., L. Sarno, and I. Tsiakas, (2011), Spot and forward volatility in foreign exchange, *Journal of Financial Economics* 100, 496 – 513.

Du, W., C. Pflueger, and J. Schreger, (2020), Sovereign debt portfolios, bond risks, and the credibility of monetary policy, *Journal of Finance* .

Duffee, G. R., (2018), Expected inflation and other determinants of treasury yields, *The Journal of Finance* 73.

Epstein, L. G. and S. Zin, (1989a), Substitution, risk aversion and the temporal behavior of consumption and asset returns: A theoretical framework, *Econometrica* 57, 937–969.

Epstein, L. G. and S. E. Zin, (1989b), Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework, *Econometrica* 57, 937–969.

Farhi, E., S. Fraiberger, X. Gabaix, R. Ranciere, and A. Verdelhan, (2015), Crash risk in currency markets, Working paper.

Farhi, E. and I. Werning, (2014), Dilemma not trilemma? Capital controls and exchange rates with volatile capital flows, *IMF Economic Review* 62, 569–605.

Froot, K. A. and T. Ramadorai, (2005), Currency returns, intrinsic value, and institutional-investor flows, *The Journal of Finance* 60.

Froot, K. A. and J. C. Stein, (1991), Exchange rates and foreign direct investment: An imperfect capital markets approach, *The Quarterly Journal of Economics* 106, 1191–1217.

Gabaix, X., (2012), Variable rare disasters: An exactly solved framework for ten puzzles in macro-finance, *Quarterly Journal of Economics* 127, 645–700.

Gabaix, X. and M. Maggiori, (2015), International liquidity and exchange rate dynamics, *Quarterly Journal of Economics* 130, 1369–420.

Gopinath, G., E. Boz, C. Casas, F. J. Díez, P.-O. Gourinchas, and M. Plagborg-Møller, (2020), Dominant currency paradigm, *American Economic Review* 110.

Gourinchas, P. O. and H. Rey, (2007), International financial adjustment, *Journal of Political Economy* 115, 665–703.

Gourio, F., (2012), Disaster risk and business cycles, *American Economic Review* 102, 2734–2766.

Gourio, F., M. Siemer, and A. Verdelhan, (2014a), Uncertainty and international capital flows, Working paper.

Gourio, F., M. Siemer, and A. Verdelhan, (2014b), Uncertainty betas and international capital flows, Working paper, MIT.

Hansen, L., (2012), Dynamic value decomposition in stochastic economies, *Econometrica* 80, 911–967.

Hassan, T., (2013), Country size, currency unions, and international asset returns, *The Journal of Finance* 68, 2269–308.

Hassan, T., T. Mertens, and T. Zhang, (2015), Currency manipulation, *Working Paper* .

Hassan, T. A., T. M. Mertens, and T. Zhang, (2016), Not so disconnected: Exchange rates and the capital stock, *Journal of International Economics* 99, S43–S57.

Heyerdahl-Larsen, C., (2015), Asset Prices and Real Exchange Rates with Deep Habits., *Review of Financial Studies* 27, 3280–317.

Jiang, Z., (2019), Fiscal cyclicality and currency risk premia Working paper, Northwestern University.

Kalemi-Ozcan, S., L. Laeven, and D. Moreno, (2020), Debt overhang, rollover risk, and corporateinvestment: Evidence from the european crisis, *Working Paper* .

Koijen, R. and M. Yogo, (2019), Exchange rates and asset prices in a global demand system, *Journal of Financial Economics* .

Le, A. and K. J. Singleton, (2010), An equilibrium term structure model with recursive preferences, *American Economic Review* 100, 557–561.

Lilley, A., M. Maggiori, B. Neiman, and J. Schreger, (2020), Exchange rate reconnect, *The Review of Economics and Statistics* forthcoming.

Lustig, H. and R. J. Richmond, (2019), Gravity in the Exchange Rate Factor Structure, *The Review of Financial Studies* 33, 3492–3540.

Lustig, H., N. Roussanov, and A. Verdelhan, (2011), Common risk factors in currency markets, *Review of Financial Studies* 24, 3731–77.

Lustig, H., N. Roussanov, and A. Verdelhan, (2014), Countercyclical currency risk premia, *Journal of Financial Economics* 111(3), 527–553.

Lustig, H., A. Stathopoulos, and A. Verdelhan, (2019), The term structure of currency carry trade risk premia, *American Economic Review* 109, 4142–4177.

Lustig, H. and A. Verdelhan, (2018), Does incomplete spanning in international financial markets help to explain exchange rates?, *Working Paper* .

Maggiori, M., (2017), Financial intermediation, international risk sharing, and reserve currencies, *American Economic Review* .

Maggiori, M., B. Neiman, and J. Schreger, (2020), International currencies and capital allocation, *Journal of Political Economy* 128.

Mueller, P., A. Stathopoulos, and A. Vedolin, (2017), International correlation risk, *Journal of Financial Economics* .

Pavlova, A. and R. Rigobon, (2007), Asset prices and exchange rates, *Review of Financial Studies* 20, 1139–1181.

Pavlova, A. and R. Rigobon, (2010), An asset-pricing view of external adjustment, *Journal of International Economics* 80, 144–156.

Pavlova, A. and R. Rigobon, (2013), International macro-finance, *Handbook of Safeguarding Global Financial Stability: Political, Social, Cultural, and Economic Theories and Models* 2, 169–176.

Piazzesi, M. and M. Schneider, (2005), Equilibrium yield curves, NBER Working paper 12609.

Ready, R., N. Roussanov, and C. Ward, (2017), Commodity trade and the carry trade: A tale of two countries, *The Journal of Finance* 72, 2629–2684.

Richmond, R., (2019), Trade network centrality and currency risk premia, *Journal of Finance* 74.

Richmond, R. and Z. Jiang, (2020), Origins of international factor structures, *Working Paper* .

Sandulescu, M., F. Trojani, and A. Vedolin, (2020), Model-free international stochastic discount factors, *Journal of Finance* forthcoming.

Schreger, J. and W. Du, (2016), Local currency sovereign risk, *Journal of Finance* .

Song, D., (2017), Bond Market Exposures to Macroeconomic and Monetary Policy Risks, *The Review of Financial Studies* 30, 2761–2817.

Stathopoulos, A., (2017), Asset prices and risk sharing in open economies, *Review of Financial Studies* .

Stock, J. H. and M. W. Watson, (2008), Phillips curve inflation forecasts, *NBER Working Paper No. w14322* .

Verdelhan, A., (2018), The share of systematic variation in bilateral exchange rates, *The Journal of Finance* 73, 375–418.

Wachter, J. A., (2006), A consumption-based model of the term structure of interest rates, *Journal of Financial Economics* 79, 365 – 399.

Zhang, S., (2020), Limited risk sharing and international equity returns, *The Journal of Finance* forthcoming.

Zviadadze, I., (2017), Term structure of consumption risk premia in the cross section of currency returns, *Journal of Finance* .

A Data Sources

A.1 Yields

We collected yields data from January 1995 to May 2018 from Datastream for the below countries and maturities. We collected yields from June 2018 through December 2018 from Datastream and Thomas Reuters Eikon.

1. Australia: 1/2/3/4/5/6/7/8/9/10/12/15/20/30 years;
2. Canada: 1/3/6 months, 1/2/3/4/5/7/10/15/20/30 years;
3. Germany: 1/3/6/9 months, 1/2/3/4/5/6/7/8/9/10/15/20/25/30 years;
4. Japan: 1/3/6/9 months, 1/2/3/4/5/6/7/8/9/10/15/20/30 years;
5. New Zealand: 1/3/6 months, 1/2/5/7/10/15/20 years;
6. Norway: 3/6/9 months, 1/2/3/5/10 years;
7. Sweden: 1/3/6 months, 2/5/7/10/15/20 years;
8. Switzerland: 1/3/6 months, 1/2/3/4/5/6/7/8/9/10/15/20/30 years;
9. United Kingdom: 1/3/6 months, 1/2/3/4/5/6/7/8/9/10/12/15/20/25/30 years;
10. United States: 1/3/6 months, 1/2/3/5/7/10/30 years.

Because the 3-month yield is central to our analysis, and Datastream does not offer Australian 3-month yields, we obtained Australian 3-month yields from the website of the Federal Reserve Bank of Saint Louis (FRED). To account for missing points on each country's term structure, we implemented spline interpolation to obtain approximate bond yields for all maturities of interest.

A.2 Exchange rates

We collected monthly exchange rate time series from the website of the Federal Reserve Bank of Saint Louis (FRED). All exchange rates are relative to the US Dollar. The specific series ids are: EXCAUS, EXJPUS, EXNOUS, EXSDUS, EXSZUS, EXUSAL, EXUSEU, EXUSNZ, EXUSUK.

A.3 Inflation forecasts

Benchmark. The data for inflation forecasts is collected from the website of the OECD, which is available at https://data.oecd.org/price/inflation-forecast.htm. The following is the description of these series reported on the OECD website: "Inflation forecast is measured in terms of the consumer price index (CPI) or harmonised index of consumer prices (HICP) for euro area countries, the euro area aggregate and the United Kingdom. Inflation measures the general evolution of prices. It is defined as the change in the prices of a basket of goods and services that are typically purchased by households. Projections are based on an assessment of the economic climate in individual countries and the world economy, using a combination of model-based analyses and expert judgement. The indicator is expressed in annual growth rates."

Additional Forecasts. We check the robustness of our findings regarding the differential degree of exposure of expected inflation of each country to global expected inflation by using alternative methodologies to forecast inflation. We follow Stock and Watson (2008) and consider two classes of models. The first one is based on estimating the following autoregressive model for inflation growth rates

$$\pi_{i,t+1} - \pi_{i,t} \;=\; \alpha_i + \beta_i(L)\left(\pi_{i,t} - \pi_{i,t-1}\right) + \varepsilon_{i,t+1}, \quad \forall i \in G10 \tag{A.1}$$

where L is the lag operator. The number of lags is selected to maximize the Akaike Information Criterion in each country. We perform the analysis by using two alternative measures of inflation: "All Items" (see panel (2) of Table 3) and "All Items Non-Food, Non-Energy" (see panel (3) of Table 3). Both sets of inflation time series are from the OECD website (`https://stats.oecd.org/OECDStat_Metadata/ShowMetadata.ashx?Dataset=PRICES_CPI`).

The second class of models is based on the Phillips curve and it amounts to estimating

$$\pi_{i,t+1} - \pi_{i,t} \;=\; \alpha_i + \beta_i(L)\left(\pi_{i,t} - \pi_{i,t-1}\right) + \gamma_i(L)\left(u_{i,t} - u_{i,t-1}\right) + \varepsilon_{i,t+1}, \quad \forall i \in G10 \tag{A.2}$$

where $u_{i,t}$ denotes the unemployment rate of country i (OECD data: `https://data.oecd.org/unemp/harmonised-unemployment-rate-hur.htm`). The number of lags for inflation and unemployment is country and variable specific and it is selected to maximize the Akaike Information Criterion. Also in this case we replicate our analysis for the two types of inflation considered above. The results for the exposure to G10 expected inflation is reported in panels (4) and (5) of Table 3.

The last class of models is a simple unit root forecasting model:

$$E_t\left[\pi_{i,t+1}\right] \;=\; \pi_{i,t}, \quad \forall i \in G10. \tag{A.3}$$

The results for the two types of inflation that we consider are in panels (6) and (7) of Table 3.

TABLE A.1: Exposures to Expected Inflation (Alternative Models)

(1) OECD Forecasts (benchmark)

	NOR	AUS	NZL	CAN	JPN	GER	US	UK	SWI	SWE
$\beta_{i,\pi}$	0.20	0.53	0.58	0.60	0.69	0.79	1.11	1.23	1.44	1.81
	(0.13)	(0.13)	(0.23)	(0.09)	(0.20)	(0.05)	(0.09)	(0.29)	(0.25)	(0.26)

(2) AR Model (Inflation, All Items)

	NOR	AUS	NZL	CAN	JPN	GER	US	UK	SWI	SWE	corr w/(1)
$\beta_{i,\pi}$	0.34	0.84	0.81	0.84	0.15	1.23	1.08	1.31	1.02	2.00	0.80
	(0.11)	(0.14)	(0.21)	(0.07)	(0.38)	(0.16)	(0.08)	(0.18)	(0.28)	(0.25)	

(3) AR Model (Inflation, All Items less Food and Energy)

	NOR	AUS	NZL	CAN	JPN	GER	US	UK	SWI	SWE	corr w/(1)
$\beta_{i,\pi}$	0.47	0.66	0.62	0.81	1.27	0.86	0.94	1.56	1.73	2.62	0.92
	(0.23)	(0.24)	(0.21)	(0.12)	(0.17)	(0.27)	(0.05)	(0.20)	(0.35)	(0.20)	

(4) Phillips curve Model (Inflation, All Items)

	NOR	AUS	NZL	CAN	JPN	GER	US	UK	SWI	SWE	corr w/(1)
$\beta_{i,\pi}$	0.25	0.77	0.72	0.84	0.35	0.71	1.38	1.10	0.63	1.58	0.74
	(0.09)	(0.11)	(0.17)	(0.08)	(0.36)	(0.06)	(0.10)	(0.14)	(0.27)	(0.11)	

(5) Phillips curve Model (Inflation, All Items less Food and Energy)

	NOR	AUS	NZL	CAN	JPN	GER	US	UK	SWI	SWE	corr w/(1)
$\beta_{i,\pi}$	0.60	0.51	0.59	0.81	1.06	0.91	0.91	1.55	1.70	2.57	0.92
	(0.13)	(0.42)	(0.23)	(0.09)	(0.12)	(0.28)	(0.06)	(0.21)	(0.32)	(0.20)	

(6) Unit Root Model (Inflation, All Items)

	NOR	AUS	NZL	CAN	JPN	GER	US	UK	SWI	SWE	corr w/(1)
$\beta_{i,\pi}$	0.32	0.80	0.77	0.84	0.77	0.81	1.06	1.37	1.45	2.09	0.97
	(0.12)	(0.14)	(0.21)	(0.08)	(0.2)	(0.14)	(0.11)	(0.24)	(0.25)	(0.28)	

(7) Unit Root Model (Inflation, All Items less Food and Energy)

	NOR	AUS	NZL	CAN	JPN	GER	US	UK	SWI	SWE	corr w/(1)
$\beta_{i,\pi}$	0.52	0.60	0.59	0.85	0.95	0.98	0.92	1.58	1.71	2.47	0.94
	(0.09)	(0.28)	(0.25)	(0.09)	(0.07)	(0.28)	(0.04)	(0.24)	(0.17)	(0.23)	

Notes - Exposures of each country's expected inflation to GDP weighted expectations of inflation. Panel (1) reports the estimates of $\beta_{i,y}$ in equation (2) of the main text using OECD forecasts (our benchmark). In panels (2) and (3), we report the exposures obtained using the forecasts based on equation (A.1) for "Total Items" and "Total Items non-Food, non-Energy" inflation. In panels (4) and (5), we report the exposures obtained using the forecasts based on equation (A.2) for "Total Items" and "Total Items non-Food, non-Energy" inflation. In panels (6) and (7), we report the exposures obtained using the forecasts based on equation (A.3) for "Total Items" and "Total Items non-Food, non-Energy" inflation. All data used in the forecasting models is annual. The numbers in parenthesis underneath each estimated coefficient are standard errors.

The estimates of the exposures of these alternative expected inflation on the global expected inflation obtained as the GDP weighted average of the corresponding inflation expectations are topically close to those obtained using OECD forecasts, as in our benchmark. The correlation of $\beta_{i,\pi}$ obtained the alternative forecasting models with our benchmark ranges from 0.74 to 0.97 (see last column of Table 3).

A.4 Real GDP growth rate forecasts

The data for real GDP forecasts is collected from the website of the OECD, which is available at https://data.oecd.org/gdp/real-gdp-forecast.htm. The following is the description of these series reported on the OECD website: "Real gross domestic product (GDP) is GDP given in constant prices and refers to the volume level of GDP. Constant price estimates of GDP are obtained by expressing values of all goods and services produced in a given year, expressed in terms of a base period. Forecast is based on an assessment of the economic climate in individual countries and the world economy, using a combination of model-based analyses and expert judgement. This indicator is measured in growth rates compared to previous year."

B Additional Empirical Results

Slope of the yield curve and global expectations. In table B.2 we analyze the relationship between the slope of the yield curve of each country and G10 expected GDP growth and expected inflation. Specifically, we estimate the two following regressions:

$$y_{i,120,t} - y_{i,3,t} = \delta_{y,0} \cdot E_t\left[\Delta y_{g10,t}\right] + \delta_{\pi,0} \cdot E_t\left[\Delta \pi_{g10,t}\right] + \varepsilon_{i,t}, \tag{B.4}$$

$$y_{i,120,t} - y_{i,3,t} = (\delta_{y,0} + \delta_{y,1} \cdot \beta_{i,y}) \cdot E_t\left[\Delta y_{g10,t}\right] + (\delta_{\pi,0} + \delta_{\pi,1} \cdot \beta_{i,\pi}) \cdot E_t\left[\Delta \pi_{g10,t}\right] + \xi_{i,t}, \tag{B.5}$$

for $i \in G10$ and where $y_{i,120,t} - y_{i,3,t}$ denotes the difference between the 10 years and 3 months yield in country i, $E_t\left[\Delta y_{g10,t}\right]$ and $E_t\left[\Delta \pi_{g10,t}\right]$ denote the expected GDP growth rate and inflation in GDP-weighted aggregate of G10 countries, and $\beta_{i,y}$ and $\beta_{i,\pi}$ are country i's exposures to expected GDP growth rate and inflation estimate in table 3 of the main text. All variables are demeaned.

Focusing on the estimates of the parameters of equation (B.4), we note that the loadings on both $E_t\left[\Delta y_{g10,t}\right]$ and $E_t\left[\Delta \pi_{g10,t}\right]$ are negative, although only the exposure to inflation ($\delta_{\pi,0}$) is statistically significant (see the row labeled "No interaction" in table B.2). When turning our attention to the estimates of equation (B.5), not only we confirm the larger impact of expected inflation on slopes, but we can also establish that the impact of expected inflation on slope is more negative for countries with large $\beta_{i,\pi}$ (see the row labeled "Interaction with $\beta_{i,y}$ and $\beta_{i,\pi}$" in table B.2).

Taken together, these findings suggest that (i) the slopes of the yield curves are more sensitive to shocks to the expected inflation, and (ii) countries with larger $\beta_{i,\pi}$ tend to respond more to news to expected inflation. Within the context of our analysis, this means that countries like the UK, Sweden, and Switzerland (i.e. high $\beta_{i,\pi}$ countries) should experience the largest increase in the slope of their yield curves following a negative shock to global expected inflation.

TABLE B.2: Slope of the Yield Curve and Expectations

	$\delta_{y,0}$	$\delta_{y,1}$	$\delta_{\pi,0}$	$\delta_{\pi,1}$
No interaction	-0.004	–	-0.373^{***}	–
	(0.005)		(0.015)	
Interaction with $\beta_{i,y}$ and $\beta_{i,\pi}$	-0.068^{***}	0.090	-0.267^{***}	-0.100^{***}
	(0.006)	(0.005)	(0.044)	(0.014)

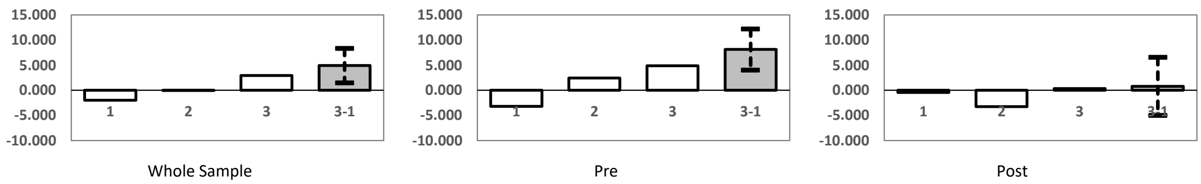
Notes - Relationship between the slope of the yield curve in each country and global expected GDP growth rate and inflation. The row labeled "No Interaction" reports the estimates of equation (B.4). The row label "Interaction with $\beta_{i,y}$ and $\beta_{i,\pi}$" reports the estimate of equation (B.5). All parameters are pooled in the cross-section of G10 countries and estimated using GMM. The numbers in parenthesis are HAC adjusted standard errors. One, two, and three stars denote statistical significance at the 10%, 5%, and 1% confidence level.

C Robustness of Empirical results

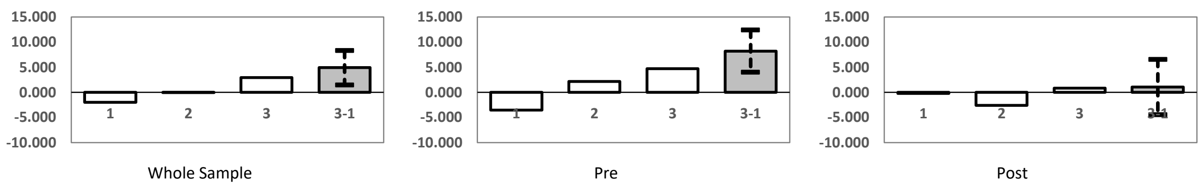
Figures C.1 and C.2 report several robustness tests for the analysis of the traditional and slope carries presented in tables 1 and 2 of the main text, respectively. In each panel, we report the average returns of each interest rate-sorted portfolio along with the spread between the two extreme portfolios.

Panel (a) reports the same exercise as in table 1 of the main text. In panel (b), we change the break-point from 7/2008 to 1/2008. Panel (c) refers to our results for the case in which we replace GDP weights to equal weights. In panel (d), we repeat the same exercise as in table 1 of the main text by trimming the series of excess returns by the 10% extreme observations. Panels (e)-(h) replicate the same findings in panels (a)-(d) using log returns (as opposed to gross returns). In panels (i)-(k), we report our findings by changing the base currency from the US Dollar to the Euro, the Yen, and the British Pound, respectively.

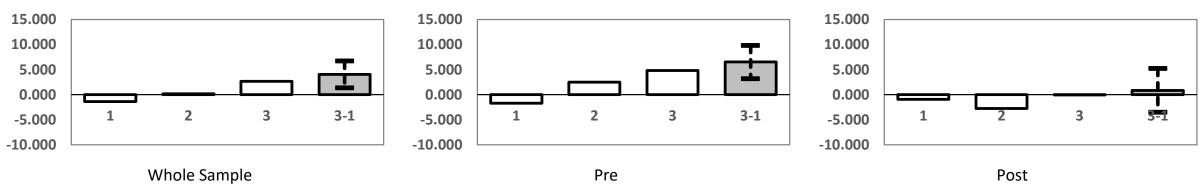
a) Gross returns, 7/2008 breakpoint



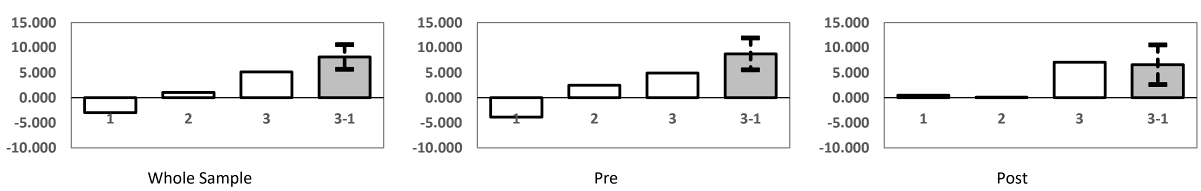
b) Gross returns, 1/2008 breakpoint



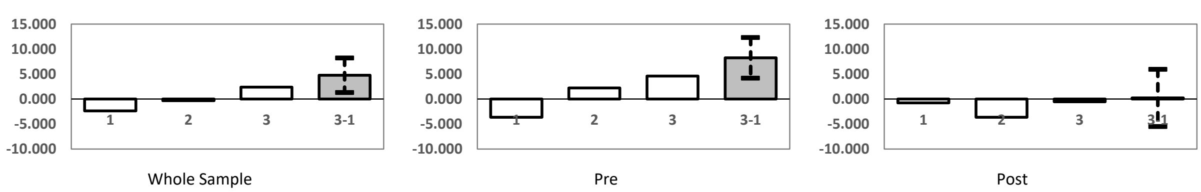
c) Gross returns, equal-weighted portfolios



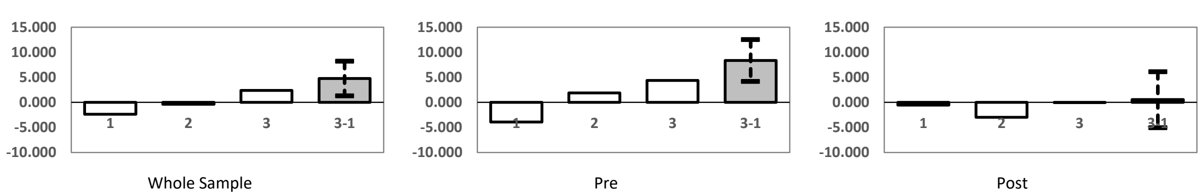
d) Gross returns, 10% winsorization



e) Log returns, 7/2008 breakpoint



f) Log returns, 1/2008 breakpoint



Continued on the next page ↪

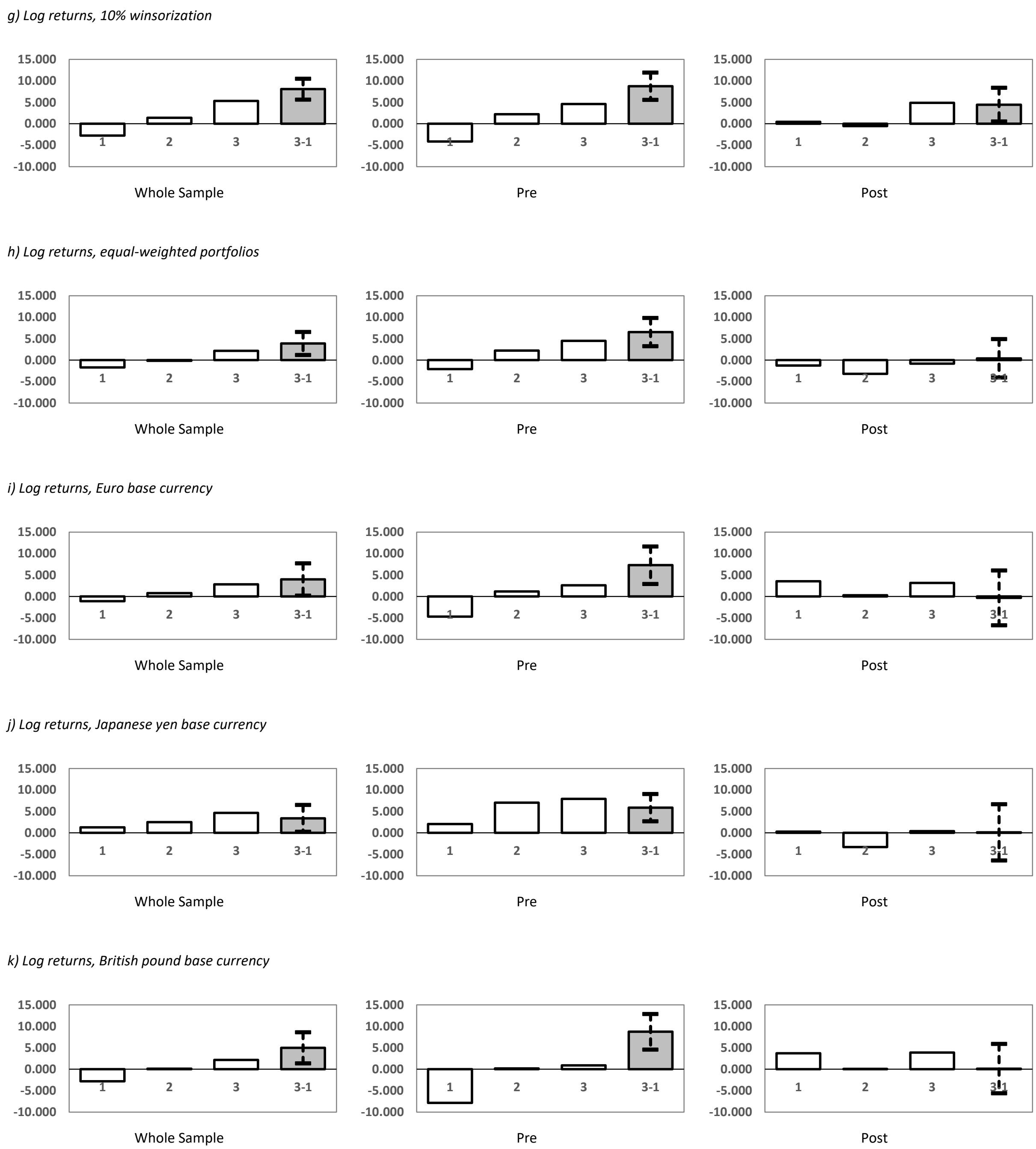
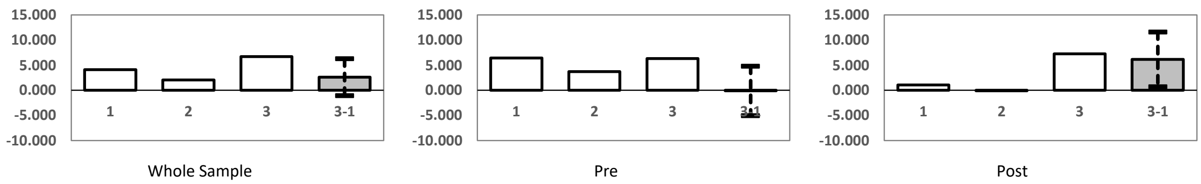
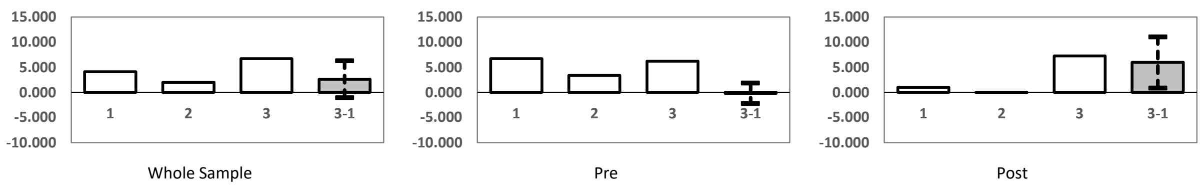


FIG. C.1 - Traditional carry: robustness. This figure reports the average returns of the portfolios sorted by the level of the 3 months interest rate. Each row of plots reports three cases: one using the entire sample, and two using each of our two sub-samples. For each case, we report the average excess returns of three portfolios sorted by increasing the level of the interest rate, as well as the difference between portfolio 3 and portfolio 1. For the latter we also report the 90% confidence interval. As a benchmark, panel (a) reports the same exercise as in table 1 of the main text. In panel (b) we change the break-point from 7/2008 to 1/2008. Panel (c) displays our results for the case in which we replace GDP weights to equal weights. In panel (d) we repeat the same exercise as in table 1 of the main text by trimming the series of excess returns by the 10% extreme observations. Panels (e)-(h) replicate the same findings in panels (a)-(d) using log (as opposed to gross) returns. In panels (i)-(k) we report our findings by changing the base currency from the US Dollar to the Euro, the Yen, and the British Pound, respectively.

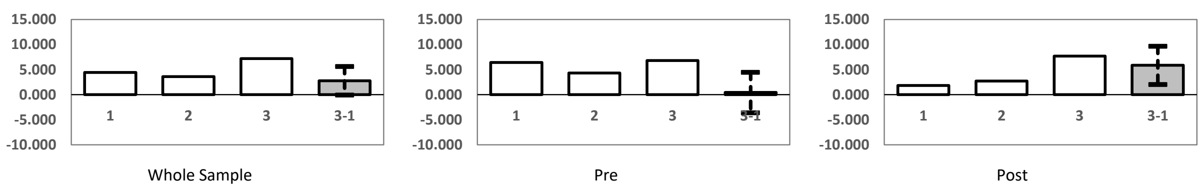
a) Gross returns, 7/2008 breakpoint



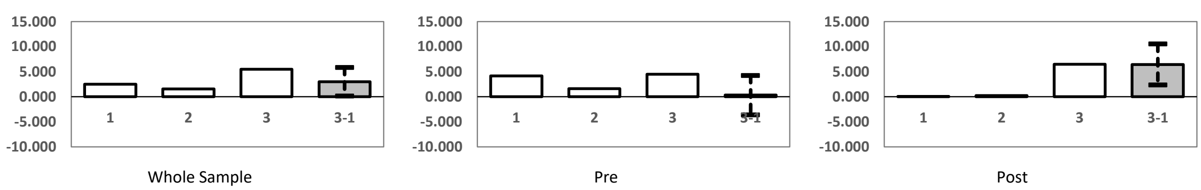
b) Gross returns, 1/2008 breakpoint



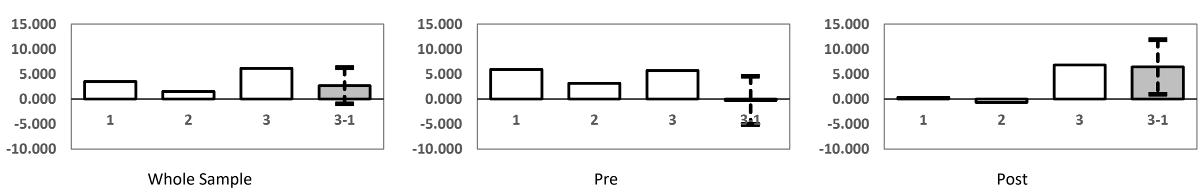
c) Gross returns, equal-weighted portfolios



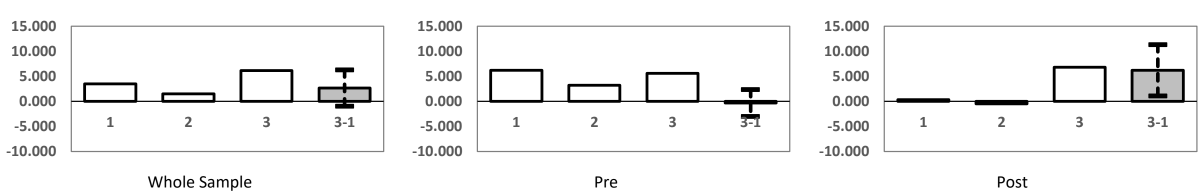
d) Gross returns, 10% winsorization



e) Log returns, 7/2008 breakpoint

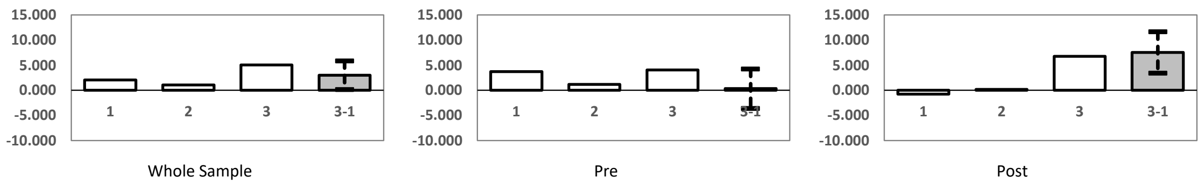


f) Log returns, 1/2008 breakpoint

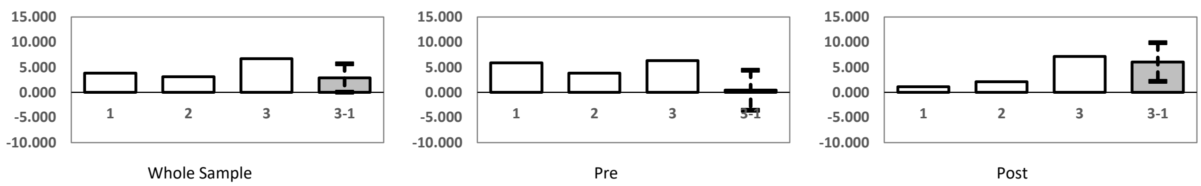


Continued on the next page ↪

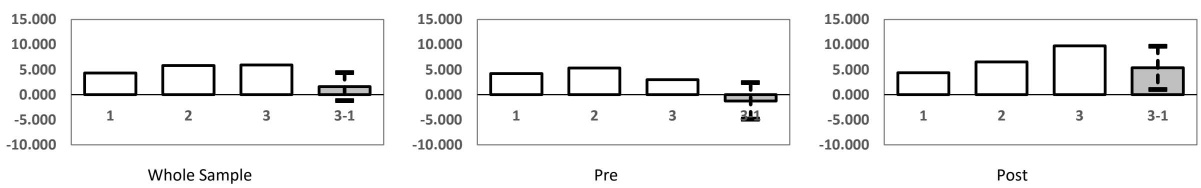
g) Log returns, 10% winsorization



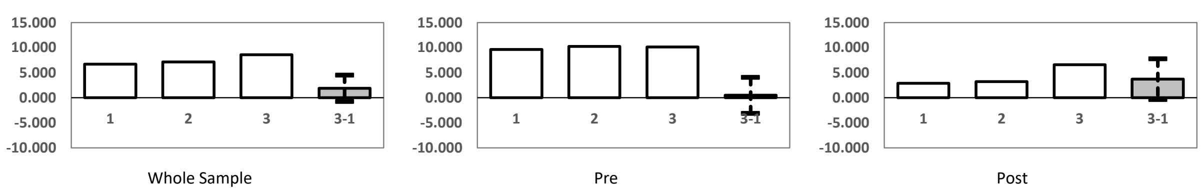
h) Log returns, equal-weighted portfolios



i) Log returns, Euro base currency, equal-weighted portfolios



j) Log returns, Japanese yen base currency, equal-weighted portfolios



k) Log returns, British pound base currency, equal-weighted portfolios

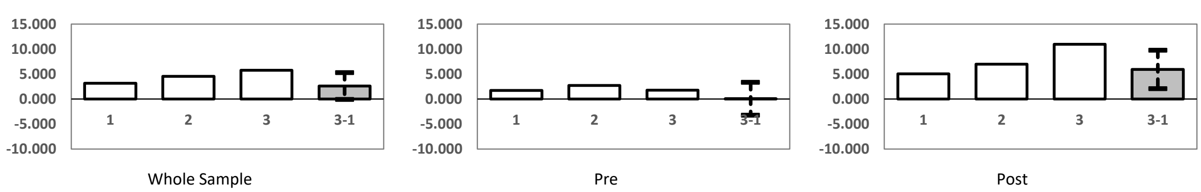


FIG. C.2 - Slope carry: robustness. This figure reports the average returns of the portfolios sorted by the slope of the yield curve. Each row of figures reports three cases: one using the entire sample, and two using each of our two sub-samples. For each case we report the average excess returns of three portfolios sorted by increasing the level of the interest rate, as well as the difference between portfolio 3 and portfolio 1. For the latter we also report the 90% confidence interval. As a benchmark, panel (a) reports the same exercise as in table 2 of the main text. In panel (b) we change the break-point from 7/2008 to 1/2008. Panel (c) displays our results for the case in which we replace GDP weights to equal weights. In panel (d) we repeat the same exercise as in table 2 of the main text by trimming the series of excess returns by the 10% extreme observations. Panels (e)-(h) replicate the same findings in panels (a)-(d) using log (as opposed to gross) returns. In panels (i)-(k) we report our findings by changing the base currency from the US Dollar to the Euro, the Yen, and the British Pound, respectively.

D Derivations of Utility, SDFs, and Bond Prices - EIS=1

D.1 Equilibrium utility

We can solve for $U_{i,t} - \log C_{i,t}$:

$$V_{i,t} \;=\; U_{i,t} - \log C_{i,t} = \delta\theta \log E_t \exp\left\{\frac{V_{i,t+1} + \Delta c_{i,t+1}}{\theta}\right\}$$

Guess $V_{i,t} = A_i + B_{i,c}x_{c,t} + B_{i,\pi}x_{\pi,t}$:

$$\begin{aligned}
V_{i,t} &= \delta\theta \log E_t \exp\left\{\frac{A_i + B_{i,c}x_{c,t+1} + B_{i,\pi}x_{\pi,t+1} + \mu_c^i + \beta_i^c x_{c,t} + \sigma_c \eta_{i,t+1}^c}{\theta}\right\} \\
&= \delta\left[A_i + \mu_c^i\right] + \delta\left[B_{i,c}\rho_c x_{c,t} + B_{i,c}\rho_{c\pi}x_{\pi,t} + B_{i,\pi}\rho_\pi x_{\pi,t} + \beta_i^c x_{c,t}\right] + \frac{\delta}{2\theta}\left[B_{i,c}^2\sigma_{x,c}^2 + B_{i,\pi}^2\sigma_{x,\pi}^2 + \sigma_c^2\right] \\
&= \delta\left[A_i + \mu_c^i + \frac{1}{2\theta}\left(B_{i,c}^2\sigma_{x,c}^2 + B_{i,\pi}^2\sigma_{x,\pi}^2 + \sigma_c^2\right)\right] + \delta\left[B_{i,c}\rho_c + \beta_i^c\right]x_{c,t} + \delta\left[B_{i,c}\rho_{c\pi} + \rho_\pi B_{i,\pi}\right]x_{\pi,t},
\end{aligned}$$

Matching coefficients yields the solution:

$$B_{i,c} \;=\; \frac{\delta\beta_i^c}{1-\delta\rho_c}, \quad B_{i,\pi} = \frac{\delta\rho_{c\pi}}{1-\delta\rho_\pi}B_{i,c}, \quad A_i = \frac{\delta}{1-\delta}\left[\mu_c^i + \frac{1}{2\theta}\left(B_{i,c}^2\sigma_{x,c}^2 + B_{i,\pi}^2\sigma_{x,\pi}^2 + \sigma_c^2\right)\right]$$

Also note that, by definition:

$$\log E_t \exp\left\{\frac{V_{i,t+1} + \Delta c_{i,t+1}}{\theta}\right\} = \frac{V_{i,t}}{\delta\theta}.$$

D.2 Real SDF

The real stochastic discount factor is:

$$\begin{aligned}
m_{i,t+1}^{real} &= \log\delta - \Delta c_{i,t+1} + \frac{U_{i,t+1}}{\theta} - \log E_t \exp\left\{\frac{U_{i,t+1}}{\theta}\right\} \\
&= \log\delta + \frac{V_{i,t+1}}{\theta} - \log E_t \exp\left\{\frac{V_{i,t+1} + \Delta c_{i,t+1}}{\theta}\right\} - \left(1 - \frac{1}{\theta}\right)\Delta c_{i,t+1}.
\end{aligned}$$

Plugging in the solution for $V_{i,t}$ and the law of motion for consumption growth:

$$\begin{aligned} m_{i,t+1}^{real} &= \log\delta + \frac{A_i + B_{i,c}x_{c,t+1} + B_{i,\pi}x_{\pi,t+1}}{\theta} - \frac{V_{i,t}}{\delta\theta} - \left(1 - \frac{1}{\theta}\right)\left(\mu_c^i + \beta_i^c x_{c,t} + \sigma_c \eta_{i,t+1}^c\right) \\ &= \bar{m}_i^{real} - \beta_c^i x_{c,t} - k_{\varepsilon c}^i \sigma_{x,c}\varepsilon_{c,t+1} + k_{\varepsilon\pi}^i \sigma_{x,\pi}\varepsilon_{\pi,t+1} - \gamma\sigma_c\eta_{i,t+1}^c \end{aligned}$$

where:

$$\begin{aligned} \bar{m}_i^{real} &= \log\delta - \frac{1}{2}(1-\gamma)^2\sigma_c^2 - \mu_c^i - \frac{1}{2}\left[(k_{\varepsilon c}^i\sigma_{x,c})^2 + ((k_{\varepsilon\pi}\sigma_{x,\pi})^2\right], \\ k_{\varepsilon c}^i &= (\gamma-1)\beta_c^i\left(\frac{\delta}{1-\delta\rho_c}\right), \\ k_{\varepsilon\pi}^i &= -\rho_{c\pi}k_{\varepsilon c}^i\left(\frac{\delta}{1-\delta\rho_\pi}\right). \end{aligned}$$

D.3 Nominal SDF

The nominal stochastic discount factor, $m_{i,t+1}$, is $m_{i,t+1}^{real} - \pi_{i,t+1}$. We have:

$$m_{i,t+1} = \bar{m}_i - \beta_c^i x_{c,t} - \beta_\pi^i x_{\pi,t} - k_{\varepsilon c}^i\sigma_{x,c}\varepsilon_{c,t+1} + k_{\varepsilon\pi}^i\sigma_{x,\pi}\varepsilon_{\pi,t+1} - \gamma\sigma_c\eta_{i,t+1}^c - \sigma_\pi\eta_{i,t+1}^\pi,$$

where $\bar{m}_i = \bar{m}_i^{real} - \mu_\pi^i$.

D.4 Permanent and Transitory components

Consider the (level) nominal stochastic factor $M_{i,t+1} = \exp(m_{i,t+1})$. Use the eigenfunction problem of Alvarez and Jermann (2005) and Hansen (2012). Let $f_{i,t+1}^{(e)}$ be the eigenfunction, then it must be the case that:

$$E_t\left[M_{i,t+1}f_{i,t+1}^{(e)}\right] = \xi_i f_{i,t}^{(e)}$$

where ξ is the eigenvalue. Accordingly the permanent and transitory components are:

$$M_{i,t+1}^P = M_{i,t+1}\frac{f_{i,t+1}^{(e)}}{\xi_i f_{i,t}^{(e)}}, \quad M_{i,t+1}^T = \frac{\xi_i f_{i,t}^{(e)}}{f_{i,t+1}^{(e)}}$$

Conjecture that $f^{(e)}_{i,t+1} = \exp\left\{f_{i,c}x_{c,t+1} + f_{i,\pi}x_{\pi,t+1}\right\}$. This implies:

$$
\begin{aligned}
\log E_t\left[M_{i,t+1}f^{(e)}_{i,t+1}\right] &= \log E_t \exp\left\{m_{i,t+1} + \log\left(f^{(e)}_{i,t+1}\right)\right\} \\
&= \log E_t \exp\Big\{\bar{m}_i - \beta^i_c x_{c,t} - \beta^i_\pi x_{\pi,t} - k^i_{\varepsilon c}\sigma_{x,c}\varepsilon_{c,t+1} + k^i_{\varepsilon\pi}\sigma_{x,\pi}\varepsilon_{\pi,t+1} - \gamma\sigma_c\eta^c_{i,t+1} \\
&\quad -\sigma_\pi\eta^\pi_{i,t+1} + f_{i,c}\left(\rho_c x_{c,t} + \rho_{c\pi}x_{\pi,t} + \sigma_{x,c}\varepsilon_{c,t+1}\right) + f_{i,\pi}\left(\rho_\pi x_{\pi,t} + \sigma_{x,\pi}\varepsilon_{\pi,t+1}\right)\Big\} \\
&= \bar{m}_i + \frac{1}{2}\left((k^i_{\varepsilon c})^2\sigma^2_{x,c} + f^2_{i,c}\sigma^2_{x,c} - 2k^i_{\varepsilon c}f_{i,c}\sigma^2_{x,c}\right) + \frac{1}{2}\left((k^i_{\varepsilon\pi})^2\sigma^2_{x,\pi} + f^2_{i,\pi}\sigma^2_{x,\pi} + 2k^i_{\varepsilon\pi}f_{i,\pi}\sigma^2_{x,\pi}\right) \\
&\quad + \frac{\gamma^2\sigma^2_c}{2} + \frac{\sigma^2_\pi}{2} - \left(\beta^c_i - f_{i,c}\rho_c\right)x_{c,t} - \left(\beta^\pi_i - f_{i,c}\rho_{c\pi} - f_{i,\pi}\rho_\pi\right)x_{\pi,t}
\end{aligned}
$$

Using the expression for $\bar{m}_i$, we have

$$
\begin{aligned}
\log E_t\left[M_{i,t+1}f^{(e)}_{i,t+1}\right] &= \left[\log\delta - \mu^i_c - \mu^i_\pi + \frac{f^2_{i,c}\sigma^2_{x,c}}{2} - k^i_{\varepsilon c}f_{i,c}\sigma^2_{x,c} + \frac{f^2_{i,\pi}\sigma^2_{x,\pi}}{2} + k^i_{\varepsilon\pi}f_{i,\pi}\sigma^2_{x,\pi} + \frac{\sigma^2_\pi}{2} + (\gamma - \frac{1}{2})\sigma^2_c\right] \\
&\quad - \left(\beta^c_i - f_{i,c}\rho_c\right)x_{c,t} - \left(\beta^\pi_i - f_{i,c}\rho_{c\pi} - f_{i,\pi}\rho_\pi\right)x_{\pi,t} \\
&= \log\xi_i + f_{i,c}x_{c,t} + f_{i,\pi}x_{\pi,t},
\end{aligned}
$$

where the last equality follows by definition. Matching coefficients:

$$
\begin{aligned}
f_{i,c} &= \frac{-\beta^c_i}{1-\rho_c}, \quad f_{i,\pi} = \frac{-\beta^\pi_i}{1-\rho_\pi} + \frac{\rho_{c\pi}}{1-\rho_\pi}\left(\frac{-\beta^c_i}{1-\rho_c}\right) \\
\log\xi_i &= \log\delta - \mu^i_c - \mu^i_\pi + \frac{f^2_{i,c}\sigma^2_{x,c}}{2} - k^i_{\varepsilon c}f_{i,c}\sigma^2_{x,c} + \frac{f^2_{i,\pi}\sigma^2_{x,\pi}}{2} + k^i_{\varepsilon\pi}f_{i,\pi}\sigma^2_{x,\pi} + \frac{\sigma^2_\pi}{2} + (\gamma - \frac{1}{2})\sigma^2_c
\end{aligned}
$$

Hence:

$$
\begin{aligned}
m^T_{i,t+1} &= \log\xi_i + f_{i,c}x_{c,t} + f_{i,\pi}x_{\pi,t} - f_{i,c}x_{c,t+1} - f_{i,\pi}x_{\pi,t+1} \\
&= \log\xi_i - \beta^c_i x_{c,t} - \beta^\pi_i x_{\pi,t} + \frac{\beta^c_i}{1-\rho_c}\sigma_{x,c}\varepsilon_{c,t+1} + \left(\frac{\beta^\pi_i}{1-\rho_\pi} + \frac{\rho_{c\pi}}{1-\rho_\pi}\frac{\beta^c_i}{1-\rho_c}\right)\sigma_{x,\pi}\varepsilon_{\pi,t+1}.
\end{aligned}
$$

The permanent component is:

$$
\begin{aligned}
m^P_{i,t+1} &= m_{i,t+1} - m^T_{i,t+1} \\
&= \bar{m}_i - \log\xi_i - \beta^i_c k^P_{\varepsilon c}\sigma_{x,c}\varepsilon_{c,t+1} - \left(\frac{\beta^i_\pi}{1-\rho_\pi} - \beta^i_c k^P_{\varepsilon\pi}\right)\sigma_{x,\pi}\varepsilon_{\pi,t+1} - \gamma\sigma_c\eta^c_{i,t+1} - \sigma_\pi\eta^\pi_{i,t+1},
\end{aligned}
$$

where

$$
k^P_{\varepsilon c} = \frac{k^i_{\varepsilon c}}{\beta^i_c} + \frac{1}{1-\rho_c}, \qquad k^P_{\varepsilon\pi} = \frac{k^i_{\varepsilon\pi}}{\beta^i_c} + \frac{-\rho_{c\pi}}{(1-\rho_\pi)(1-\rho_c)}.
$$

D.5 Nominal risk-free rate

The nominal risk-free rate is:

$$\begin{aligned} r^1_{i,t} &= -\log E_t \exp\{m_{i,t+1}\} \\ &= -\bar{m}_i + \beta^c_i x_{c,t} + \beta^\pi_i x_{\pi,t} - \frac{1}{2}\left(\left(k^i_{\varepsilon c}\sigma_{x,c}\right)^2 + \left(k^i_{\varepsilon\pi}\sigma_{x,\pi}\right)^2 + \gamma^2\sigma^2_c + \sigma^2_\pi\right) \\ &= \bar{r}_i + \beta^c_i x_{c,t} + \beta^\pi_i x_{\pi,t}, \end{aligned}$$

where $\bar{r}_i = \left(\mu^i_c + \mu^i_\pi\right) - \log\delta - \left(\frac{1}{2} - \frac{1}{\theta}\right)\sigma^2_c - \frac{1}{2}\sigma^2_\pi$.

D.6 Infinite maturity yields

Nominal bonds. To obtain the yields of a bond with maturity n, it is convenient to rewrite the nominal stochastic discount factor as:

$$m_{i,t+1} = -r_{i,t} - \frac{1}{2}\Lambda'_i\Lambda_i - \Lambda'_i\varepsilon_{i,t+1},$$

where

$$\varepsilon_{i,t+1} = \left[\varepsilon^\pi_{t+1}, \varepsilon^c_{i,t+1}, \eta^\pi_{i,t+1}, \eta^c_{t+1}\right]', \quad \Lambda_i = \left[-k^i_{\varepsilon\pi}\sigma_{x,\pi}, k^i_{\varepsilon c}\sigma_{x,c}, \gamma\sigma_c, \sigma_\pi\right]'$$

and the one period risk-free rate is $r_{i,t} = \bar{r}_i + \beta'_i \cdot x_t$, with $\beta'_i = [\beta^i_\pi, \beta^i_c]$ and $x'_t = [x_{\pi,t}, x_{c,t}]$. We can write the law of motion of x_t in matrix form as:

$$x_{t+1} = Kx_t + \Sigma\varepsilon_{t+1},$$

where

$$K = \begin{bmatrix} \rho_\pi & 0 \\ \rho_{c\pi} & \rho_c \end{bmatrix}, \quad \Sigma = \begin{bmatrix} \sigma_{x,\pi} & 0 & 0 & 0 \\ 0 & \sigma_{x,c} & 0 & 0 \end{bmatrix}.$$

The yield on an n period maturity bond is:

$$r^n_{i,t} = A^n_i + B^{n'}_i \cdot x_t,$$

where

$$\begin{aligned} A^n_i &= \bar{r}_i - \frac{1}{n}\sum_{j=1}^{n}\left(j \cdot B^j_i\right)' \frac{\Sigma\Sigma'}{2}\left(j \cdot B^j_i\right) - \frac{1}{n}\left(\sum_{j=1}^{n-1} j \cdot B^{j'}_i\right)\Sigma\Lambda_i \\ B^n_i &= \frac{1}{n}\beta'_i\sum_{i=0}^{n-1}K^i = \frac{1}{n}\beta'_i\left(I-K\right)^{-1}\left(I-K^n\right). \end{aligned}$$

Infinite maturity yield. By letting $n \to \infty$, it is possible to show that:

$$\begin{aligned}
\lim_{n\to\infty} B_i^n &= 0 \\
\lim_{n\to\infty} A_i^n &= \bar{r}_i - \beta_i'(I-K)^{-1}\Sigma\left[\frac{\Sigma'}{2}\left[(I-K)^{-1}\right]'\beta_i + \Lambda_i\right]
\end{aligned}$$

It follows that the yield on the infinite maturity bond is constant and equal to $r_i^\infty = \lim_{n\to\infty} A_i^n$. Specifically, we have

$$\lim_{n\to\infty} B_i^n = \lim_{n\to\infty}\frac{1}{n}\beta_i'\left(I-K\right)^{-1}\left(I-K^n\right) = 0,$$

and

$$\begin{aligned}
\lim_{n\to\infty} A_i^n &= \bar{r}_i - \lim_{n\to\infty}\frac{1}{n}\beta_i'\left[I + (I+K) + \ldots + \left(I + K + \cdots + K^{n-2}\right)\right]\Sigma\Lambda_i \\
&\quad - \lim_{n\to\infty}\frac{1}{n}\beta_i'\left[I\frac{\Sigma\Sigma'}{2}I + (I+K)\frac{\Sigma\Sigma'}{2}(I+K)' + \ldots\right]\beta_i \\
&= \bar{r}_i - \left(\lim_{n\to\infty}\frac{1}{n}\beta_i'\left[\sum_{i=0}^{n-2}\sum_{j=0}^{i}K^j\right]\Sigma\Lambda_i\right) - \left(\beta_i'(I-K)^{-1}\frac{\Sigma\Sigma'}{2}\left[(I-K)^{-1}\right]'\beta_i\right) \\
&= \bar{r}_i - \left(\lim_{n\to\infty}\frac{1}{n}\beta_i'\left[\sum_{i=0}^{n-2}(I-K)^{-1}\left(I-K^{i+1}\right)\right]\Sigma\Lambda_i\right) - \left(\beta_i'(I-K)^{-1}\frac{\Sigma\Sigma'}{2}\left[(I-K)^{-1}\right]'\beta_i\right) \\
&= \bar{r}_i - \left(\lim_{n\to\infty}\frac{n-1}{n}\beta_i'\left[(I-K)^{-1}\right]\Sigma\Lambda_i\right) + \left(\lim_{n\to\infty}\frac{1}{n}\beta_i'\left[\sum_{i=0}^{n-2}\left(I-K^{i+1}\right)\right]\Sigma\Lambda_i\right) \\
&\quad - \left(\beta_i'(I-K)^{-1}\frac{\Sigma\Sigma'}{2}\left[(I-K)^{-1}\right]'\beta_i\right) \\
&= \bar{r}_i - \left(\beta_i'\left[(I-K)^{-1}\right]\Sigma\Lambda_i\right) - (0) - \left(\beta_i'(I-K)^{-1}\frac{\Sigma\Sigma'}{2}\left[(I-K)^{-1}\right]'\beta_i\right) \\
&= \bar{r}_i - \beta_i'(I-K)^{-1}\Sigma\left[\frac{\Sigma'}{2}\left[(I-K)^{-1}\right]'\beta_i + \Lambda_i\right].
\end{aligned}$$

E FX strategies

In this section, let us use the index b to denote the base country. Without loss of generality, we impose that $\beta_c^b = \beta_\pi^b = 1$, i.e. the base country has an exposure to global growth and inflation shocks that is normalized to one. Let us also introduce the following coefficients to capture common terms in our SDFs: $k_{\varepsilon c} := k_{\varepsilon c}^i/\beta_c^i$, and $k_{\varepsilon\pi} := k_{\varepsilon\pi}^i/\beta_c^i$.

E.1 Traditional carry

The expected excess return of a strategy that is short the risk-free rate of the base country and long the short-term rate of country i is:

$$
\begin{aligned}
\log E_t\left[RX^1_{i,t+1}\right] &= \log E_t \exp\left\{-r^b_{1,t} + r^i_{1,t} + \Delta e_{bi,t+1}\right\} \\
&= V_t\left[m^b_{t+1}\right] - cov_t\left[m^b_{t+1}, m^i_{t+1}\right] = V_t\left[m^b_{t+1}\right] - \beta^i_c(k^2_{\varepsilon c}\sigma^2_{x,c} + k^2_{\varepsilon\pi}\sigma^2_{x,\pi}),
\end{aligned}
$$

implying that all of the cross sectional heterogeneity in risk premia is driven solely by β^i_c. In this case, β^i_π is irrelevant because short-term rates are not exposed to inflation news shocks risk.

Take two countries $j \in \{H, L\}$. A traditional carry strategy is long the currency of high-interest rate (**H**) countries (β^L_c) and short the currency of the low-interest rate (**L**) countries (β^H_c). The resulting traditional carry risk premium, $E[carry^T]$, is:

$$
\begin{aligned}
E[carry^T] &:= \log E_t\left[RX^1_{\mathbf{H},t+1}\right] - \log E_t\left[RX^1_{\mathbf{L},t+1}\right] \\
&= \left(\beta^H_c - \beta^L_c\right)\left[k^2_{\varepsilon c}\sigma^2_{x,c} + k^2_{\varepsilon\pi}\sigma^2_{x,\pi}\right].
\end{aligned}
$$

E.2 Slope carry

Let $hpr^\infty_{i,t+1}$ denote the holding-period return of an infinite maturity bond in country i. The conditional return associated to borrowing at the short-rate in the base country and investing for one period in the infinite maturity bond of country i is

$$
\begin{aligned}
\log E_t\left[RX^\infty_{i,t+1}\right] &= \log E_t \exp\{-r_{b,t} + hpr^\infty_{i,t+1} + \Delta e^{bi}_{t+1}\} \\
&= \log E_t \exp\{-r_{b,t} + hpr^\infty_{i,t+1} + m_{i,t+1} - m_{b,t+1}\} \\
&= \log E_t \exp\{-r_{i,t} - m^T_{i,t+1} + m^T_{i,t+1} + m^P_{i,t+1} - m_{b,t+1}\},
\end{aligned}
$$

where the second equality follows from the decomposition of the SDF of country i into its permanent and transitory components and from observing that the transitory component is equal to the opposite of the holding period return on the infinite maturity bond. It follows immediately that:

$$
\begin{aligned}
\log E_t\left[RX^\infty_{i,t+1}\right] &= \log E_t \exp\{-r_{b,t} + hpr^\infty_{i,t+1} + \Delta e^{bi}_{t+1}\} \\
&= V_t m_{b,t+1} + E_t m^P_{i,t+1} + \frac{1}{2} V_t m^P_{i,t+1} - cov_t(m^P_{i,t+1}, m_{b,t+1}) \\
&= V_t m_{b,t+1} - cov_t(m^P_{i,t+1}, m_{b,t+1})
\end{aligned}
$$

where the second equality follows directly from observing that the permanent component of the SDF is a martingale, i.e. $E_t \exp\left\{m^P_{i,t+1}\right\} = 1$. After normalizing the coefficients of the base country so that $\beta_c^b = \beta_\pi^b = 1$, we have:

$$\log E_t\left[RX_{i,t+1}^{\infty}\right] = \log E_t\left[RX_{i,t+1}^{1}\right] - \beta_i^c\left[\frac{k_{\varepsilon c}\sigma_{x,c}^2}{1-\rho_c} - \frac{\rho_{c\pi}k_{\varepsilon\pi}\sigma_{x,\pi}^2}{(1-\rho_c)(1-\rho_\pi)}\right] + \beta_i^\pi\frac{k_{\varepsilon\pi}\sigma_{x,\pi}^2}{1-\rho_\pi}.$$

Take two countries $j \in \{S, F\}$. An unconventional carry strategy is long the country of a steep **(S)** yield curve and short the country of a flat **(F)** yield curve has the following risk premium:

$$\begin{aligned}
\log E_t\left[RX_{\mathbf{S},t+1}^{\infty}\right] - \log E_t\left[RX_{\mathbf{F},t+1}^{\infty}\right] \quad &= \quad \left(\beta_c^F - \beta_c^S\right)\left[k_{\varepsilon c}^2\sigma_{x,c}^2 + k_{\varepsilon\pi}^2\sigma_{x,\pi}^2\right] \\
&+ \quad \left(\beta_c^F - \beta_c^S\right)\left[\frac{k_{\varepsilon c}\sigma_{x,c}^2}{1-\rho_c} - \frac{\rho_{c\pi}k_{\varepsilon\pi}\sigma_{x,\pi}^2}{(1-\rho_c)(1-\rho_\pi)}\right] \\
&- \quad \left(\beta_\pi^F - \beta_\pi^S\right)\frac{k_{\varepsilon\pi}\sigma_{x,\pi}^2}{1-\rho_\pi}.
\end{aligned}$$

E.3 Additional Calibration Table

Table E.1 shows our calibration for the exposure coefficients, β_c^i and β_π^i, of our 10 countries. Our values are consistent with our empirical confidence intervals.

TABLE E.1: Heterogeneous Exposure Coefficients

	Growth Risk (β_c^i)			Inflation Risk (β_π^i)		
Country	**Model**	**(95% C.I.)**		**Model**	**(95% C.I.)**	
AUS	0.48	0.25	0.65	0.87	0.27	0.79
CAN	0.90	0.75	1.13	0.50	0.42	0.77
GER	0.50	0.69	1.32	1.30	0.70	0.89
JPN	0.80	1.04	1.90	0.50	0.29	1.10
NOR	0.78	0.22	0.74	0.52	−0.05	0.45
NZL	0.57	−0.14	0.72	1.00	0.12	1.05
SWE	0.48	0.21	0.93	1.45	1.30	2.32
SWI	0.47	0.36	0.71	1.30	0.94	1.44
UK	0.50	0.44	1.03	1.50	0.65	1.82
US	1.00	0.81	1.07	1.00	0.94	1.28

Notes - This table reports our calibration of the exposure coefficients to global consumption growth and inflation risk for each country in our model. We also report their empirical confidence interval obtained following the methods described in section 2.

In table E.2, we report the unconditional averages of both the nominal risk-rates and the slope of the yield curve across our 10 countries.

TABLE E.2: Unconditional Levels and Slopes

Country	β_c	β_π	**Short Rate**	**Slope**
AUS	0.48	0.87	4.94	1.34
CAN	0.90	0.50	3.75	1.33
GER	0.50	1.30	5.33	1.39
JPN	0.80	0.50	3.95	1.29
NOR	0.78	0.52	4.00	1.36
NZL	0.47	1.00	5.09	1.32
SWE	0.48	1.45	5.52	1.05
SWI	0.47	1.30	5.39	1.16
UK	0.50	1.50	5.53	1.15
US	1.00	1.00	4.06	3.75

Notes - This table reports the unconditional mean of the short-term rate and the yield curve slope for our 10 countries.

F Derivations with $IES \neq 1$ and Demand Shocks

F.1 Preference and state variables

The preferences of the representative agent in country i are specified as

$$U_{i,t} = \left\{ (1-\delta)\Lambda_{i,t} C_{i,t}^{1-\frac{1}{\psi}} + \delta E_t \left[U_{i,t+1}^{1-\gamma} \right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\psi}}},$$

where $\Lambda_{i,t}$ is a process that captures a demand shifter. Consumption, inflation, and demand shifter follow the processes described below:

$$\begin{aligned}
\pi_{i,t+1} &= \mu_\pi^i + \beta_i^\pi x_{\pi,t} + \sigma_\pi \eta_{i,t+1}^\pi \\
\Delta c_{i,t+1} &= \mu_c^i + \beta_i^c x_{c,t} + \sigma_c \eta_{i,t+1}^c \\
\log(\Lambda_{i,t+1}/\Lambda_{i,t}) = \Delta\lambda_{i,t+1} &= x_{d,t} + \sigma_d \eta_{i,t+1}^d \\
x_{\pi,t} &= \rho_\pi x_{\pi,t-1} + \sigma_{x,\pi}\varepsilon_{\pi,t} \\
x_{c,t} &= \rho_{c\pi} x_{\pi,t-1} + \rho_c x_{c,t-1} + \sigma_{x,c}\varepsilon_{c,t} \\
x_{d,t} &= \rho_d x_{d,t-1} + \sigma_{x,d}\varepsilon_{d,t}
\end{aligned}$$

where

$$\varepsilon_{i,t} \equiv \left[\begin{array}{cccccc} \eta_{i,t}^\pi, & \eta_{i,t}^c, & \eta_{i,t}^d, & \varepsilon_{\pi,t}, & \varepsilon_{c,t}, & \varepsilon_{d,t} \end{array} \right]' \sim N(0,\Sigma)$$

All diagonal entries of Σ are equal to one. We allow for non-zero correlation between different shocks.

F.2 Return on consumption claim

The real SDF is

$$m_{i,t+1}^{\text{real}} = \theta \log\delta + \theta\Delta\lambda_{t+1} - \frac{\theta}{\psi}\Delta c_{i,t+1} + (\theta-1) r_{i,t+1}^c$$

where $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$ and $r_{i,t}^c$ is return on consumption claim. We derive the expression of $r_{i,t}^c$ and SDF using the no-arbitrage condition $E_t[M_{i,t+1} R_{i,t+1}^c] = 1$, i.e.,

$$E_t \left[\exp\left(\theta\log\delta + \theta\Delta\lambda_{t+1} - \frac{\theta}{\psi}\Delta c_{i,t+1} + \theta r_{i,t+1}^c \right) \right] = 1$$

The returns on the consumption claim can be approximated written as follows

$$r^c_{i,t+1} = \kappa_0 + \kappa_c pc_{i,t+1} - pc_{i,t} + \Delta c_{i,t+1},$$

where $pc_{i,t}$ is the log price-consumption ratio, and

$$\kappa_c = \frac{e^{\bar{pc}_i}}{1+e^{\bar{pc}_i}}, \quad \kappa_0 = \log(1+e^{\bar{pc}_i}) - \kappa_c \bar{pc}_i.$$

We conjecture that the log price-consumption ratio is a linear function of the state variables

$$pc_{i,t} = \bar{pc}_i + A_{i,1}x_{c,t} + A_{i,2}x_{\pi,t} + A_{i,3}x_{d,t}$$

Thus

$$\begin{aligned}
r^c_{i,t+1} &= \kappa_0 + \kappa_c pc_{i,t+1} - pc_{i,t} + \Delta c_{i,t+1} \\
&= \kappa_0 + \kappa_c(\bar{pc}_i + A_{i,1}x_{c,t+1} + A_{i,2}x_{\pi,t+1} + A_{i,3}x_{d,t+1}) \\
&\quad -(\bar{pc}_i + A_{i,1}x_{c,t} + A_{i,2}x_{\pi,t} + A_{i,3}x_{d,t}) + \Delta c_{i,t+1} \\
&= \kappa_0 + (\kappa_c - 1)\bar{pc}_i + A_{i,1}\kappa_c\left(\rho_{c\pi}x_{\pi,t} + \rho_c x_{c,t} + \sigma_{x,c}\varepsilon_{c,t+1}\right) + A_{i,2}\kappa_c\left(\rho_\pi x_{\pi,t} + \sigma_{x,\pi}\varepsilon_{\pi,t+1}\right) \\
&\quad +A_{i,3}\kappa_c\left(\rho_d x_{d,t} + \sigma_{x,d}\varepsilon_{d,t+1}\right) - A_{i,1}x_{c,t} - A_{i,2}x_{\pi,t} - A_{i,3}x_{d,t} + \Delta c_{i,t+1} \\
&= \bar{r}^c_i + B_{i,1}x_{c,t} + B_{i,2}x_{\pi,t} + B_{i,3}x_{d,t} + K_{i,1}\varepsilon_{i,t+1} + \Delta c_{i,t+1}
\end{aligned}$$

where

$$\begin{aligned}
\bar{r}^c_i &= \kappa_0 + (\kappa_c - 1)\bar{pc}_i \\
B_{i,1} &= A_{i,1}\kappa_c\rho_c - A_{i,1} \\
B_{i,2} &= A_{i,1}\kappa_c\rho_{c\pi} + A_{i,2}\kappa_c\rho_\pi - A_{i,2} \\
B_{i,3} &= A_{i,3}\kappa_c\rho_d - A_{i,3} \\
K_{i,1} &= \begin{bmatrix} 0, & 0, & 0, & \kappa_c A_{i,2}\sigma_{x,\pi}, & \kappa_c A_{i,1}\sigma_{x,c}, & \kappa_c A_{i,3}\sigma_{x,d} \end{bmatrix}
\end{aligned}$$

Now

$$
\begin{aligned}
1 &= E_t\left[\exp\left(\theta\log\delta + \theta\Delta\lambda_{t+1} - \frac{\theta}{\psi}\Delta c_{i,t+1} + \theta r^c_{i,t+1}\right)\right] \\
&= E_t\left[\exp\left(\theta\log\delta + \theta\Delta\lambda_{t+1} - (\frac{\theta}{\psi} - \theta)\Delta c_{i,t+1} + \theta\left(\bar{r}^c_i + B_{i,1}x_{c,t} + B_{i,2}x_{\pi,t} + B_{i,3}x_{d,t} + K_{i,1}\varepsilon_{i,t+1}\right)\right)\right] \\
&= E_t\Bigg[\exp\Bigg(\theta\log\delta + \theta x_{d,t} + \theta\sigma_d\eta^d_{i,t+1} - (\gamma-1)\left(\mu^i_c + \beta^c_i x_{c,t} + \sigma_c\eta^c_{i,t+1}\right) + ... \\
&\quad +\theta\left(\bar{r}^c_i + B_{i,1}x_{c,t} + B_{i,2}x_{\pi,t} + B_{i,3}x_{d,t} + K_{i,1}\varepsilon_{i,t+1}\right)\Bigg)\Bigg] \\
&= E_t\Bigg[\exp\Bigg(\theta\log\delta - (\gamma-1)\mu^i_c + \theta\bar{r}^c_i + \left(-(\gamma-1)\beta^c_i + \theta B_{i,1}\right)x_{c,t} + ... \\
&\quad +\theta B_{i,2}x_{\pi,t} + \theta(B_{i,3}+1)x_{d,t} + K_{i,2}\varepsilon_{i,t+1}\Bigg)\Bigg]
\end{aligned}
$$

where

$$
K_{i,2} = \begin{bmatrix} 0, & -(\gamma-1)\sigma_c, & \theta\sigma_d, & \theta\kappa_c A_{i,2}\sigma_{x,\pi}, & \theta\kappa_c A_{i,1}\sigma_{x,c}, & \theta\kappa_c A_{i,3}\sigma_{x,d} \end{bmatrix}
$$

Note that this equation holds for any realization of $x_{c,t}$ and $x_{\pi,t}$, implying that:

$$
\begin{aligned}
-(\gamma-1)\beta^c_i + \theta B_{i,1} &= 0 \\
\theta B_{i,2} &= 0 \\
\theta(B_{i,3}+1) &= 0.
\end{aligned}
$$

As a result, we obtain

$$
\begin{aligned}
B_{i,1} &= \frac{\gamma-1}{\theta}\beta^c_i = -\left(1 - \frac{1}{\psi}\right)\beta^c_i \\
B_{i,2} &= 0 \\
B_{i,3} &= -1.
\end{aligned}
$$

The solution for $A_{i,1}$, $A_{i,2}$, and $A_{i,3}$ is:

$$
A_{i,1} = \frac{(1-\gamma)\beta^c_i}{\theta(1-\kappa_c\rho_c)} = \frac{(1-\frac{1}{\psi})\beta^c_i}{1-\kappa_c\rho_c}
$$

$$
A_{i,2} = \frac{A_{i,1}\kappa_c\rho_{c\pi}}{1-\kappa_c\rho_\pi}.
$$

$$
A_{i,3} = \frac{1}{1-\kappa_c\rho_d}
$$

Given these results, the equation to pin down the value of κ_c is

$$1 = E_t\left[\exp\left(\theta \log\delta - (\gamma-1)\mu_c^i + \theta\bar{r}_i^c + K_{i,2}\varepsilon_{i,t+1}\right)\right],$$

which can be written as

$$\theta\log\delta - (\gamma-1)\mu_c^i + \theta\bar{r}_i^c + \frac{1}{2}K_{i,2}\Sigma K_{i,2}' = 0.$$

Since $\bar{r}_i^c = \kappa_0 + (\kappa_c - 1)\bar{pc}_i = \log\left(\frac{1+e^{\bar{pc}_i}}{e^{\bar{pc}_i}}\right) = -\log\kappa_c$, the above equation can be simplified and rewritten as:

$$\log\kappa_c = \log\delta + \left(1-\frac{1}{\psi}\right)\mu_c^i + \frac{1}{2\theta}K_{i,2}\Sigma K_{i,2}',$$

where

$$\begin{aligned} K_{i,2} &= \begin{bmatrix} 0, & -(\gamma-1)\sigma_c, & \theta\sigma_d, & \theta\kappa_c A_{i,2}\sigma_{x,\pi}, & \theta\kappa_c A_{i,1}\sigma_{x,c}, & \theta\kappa_c A_{i,3}\sigma_{x,d} \end{bmatrix} \\ &= \begin{bmatrix} 0, & -(\gamma-1)\sigma_c, & \theta\sigma_d, & \frac{\kappa_c\rho_{c\pi}}{1-\kappa_c\rho_\pi}\frac{(1-\gamma)\beta_i^c}{1-\kappa_c\rho_c}\kappa_c\sigma_{x,\pi}, & \frac{(1-\gamma)\beta_i^c}{1-\kappa_c\rho_c}\kappa_c\sigma_{x,c}, & \frac{\theta}{1-\kappa_c\rho_d}\kappa_c\sigma_{x,d} \end{bmatrix} \end{aligned}$$

We conclude this section by showing the expression for the consumption claim returns:

$$\begin{aligned} r_{i,t+1}^c &= \bar{r}_i^c + B_{i,1}x_{c,t} + B_{i,2}x_{\pi,t} + B_{i,3}x_{d,t} + K_{i,1}\varepsilon_{i,t+1} + \Delta c_{i,t+1} \\ &= -\log\kappa_c - \left(1-\frac{1}{\psi}\right)\beta_i^c x_{c,t} - x_{d,t} + K_{i,1}\varepsilon_{i,t+1} + \Delta c_{i,t+1}. \end{aligned}$$

F.3 Real SDF, Nominal SDF, and Risk-free Rate

The real SDF is

$$\begin{aligned} m_{i,t+1}^{\text{real}} &= \theta\log\delta + \theta\Delta\lambda_{t+1} - \frac{\theta}{\psi}\Delta c_{i,t+1} + (\theta-1)r_{i,t+1}^c \\ &= \theta\log\delta + \theta\Delta\lambda_{t+1} - \left(\frac{\theta}{\psi} - \theta + 1\right)\Delta c_{i,t+1} \\ &\quad +(\theta-1)\left(-\log\kappa_c - \left(1-\frac{1}{\psi}\right)\beta_i^c x_{c,t} - x_{d,t} + K_{i,1}\varepsilon_{i,t+1}\right) \\ &= \theta\log\delta + \theta\left(x_{d,t} + \sigma_d\eta_{i,t+1}^d\right) - \gamma(\mu_c^i + \beta_i^c x_{c,t} + \sigma_c\eta_{i,t+1}^c) \\ &\quad +(\theta-1)\left(-\log\kappa_c - \left(1-\frac{1}{\psi}\right)\beta_i^c x_{c,t} - x_{d,t} + K_{i,1}\varepsilon_{i,t+1}\right) \\ &= \bar{m}_i^{\text{real}} - \frac{1}{\psi}\beta_i^c x_{c,t} + x_{d,t} + K_{i,3}\varepsilon_{i,t+1} \end{aligned}$$

where

$$\bar{m}_i^{\text{real}} = \theta \log \delta - \gamma \mu_c^i - (\theta - 1) \log \kappa_c$$

$$\begin{aligned} K_{i,3} &= \begin{bmatrix} 0, & -\gamma\sigma_c, & \theta\sigma_d, & (\theta-1)\kappa_c A_{i,2}\sigma_{x,\pi}, & (\theta-1)\kappa_c A_{i,1}\sigma_{x,c}, & (\theta-1)\kappa_c A_{i,3}\sigma_{x,d} \end{bmatrix} \\ &= \begin{bmatrix} 0, & -\gamma\sigma_c, & \theta\sigma_d, & \frac{\rho_{c\pi}}{1-\kappa_c\rho_\pi}\frac{\frac{1}{\psi}-\gamma}{1-\kappa_c\rho_c}\beta_i^c\kappa_c^2\sigma_{x,\pi}, & \frac{\frac{1}{\psi}-\gamma}{1-\kappa_c\rho_c}\beta_i^c\kappa_c\sigma_{x,c}, & \frac{(\theta-1)\kappa_c}{1-\kappa_c\rho_d}\sigma_{x,d} \end{bmatrix} \end{aligned}$$

The nominal SDF is $m_{i,t+1} = m_{i,t+1}^{\text{real}} - \pi_{i,t+1}$, thus

$$m_{i,t+1} = \bar{m}_i - \frac{1}{\psi}\beta_i^c x_{c,t} - \beta_i^\pi x_{\pi,t} + x_{d,t} + K_{i,4}\varepsilon_{i,t+1}$$

where $\bar{m}_i = \bar{m}_i^{\text{real}} - \mu_i^\pi$, and

$$K_{i,4} = \begin{bmatrix} -\sigma_\pi, & -\gamma\sigma_c, & \theta\sigma_d, & \frac{\rho_{c\pi}}{1-\kappa_c\rho_\pi}\frac{\frac{1}{\psi}-\gamma}{1-\kappa_c\rho_c}\beta_i^c\kappa_c^2\sigma_{x,\pi}, & \frac{\frac{1}{\psi}-\gamma}{1-\kappa_c\rho_c}\beta_i^c\kappa_c\sigma_{x,c}, & \frac{(\theta-1)\kappa_c}{1-\kappa_c\rho_d}\sigma_{x,d} \end{bmatrix}$$

The nominal risk-free rate is

$$\begin{aligned} r_{i,t} &= -\log E_t \exp(m_{i,t+1}) \\ &= -\bar{m}_i + \frac{1}{\psi}\beta_i^c x_{c,t} + \beta_i^\pi x_{\pi,t} - x_{d,t} - \frac{1}{2}K_{i,4}\Sigma K_{i,4}' \\ &= \bar{r}_i + \frac{1}{\psi}\beta_i^c x_{c,t} + \beta_i^\pi x_{\pi,t} - x_{d,t} \end{aligned}$$

where

$$\bar{r}_i = -\theta \log \delta + \gamma \mu_c^i + \mu_\pi^i + (\theta - 1)\log \kappa_c - \frac{1}{2}K_{i,4}\Sigma K_{i,4}'$$

As a result, the SDF can written as

$$m_{i,t+1} = -r_{i,t} - \frac{1}{2}K_{i,4}\Sigma K_{i,4}' + K_{i,4}\varepsilon_{i,t+1}$$

and the risk-free rate

$$r_{i,t} = \bar{r}_i + \beta_i' \cdot x_t$$

where $\beta_i' = \left[\beta_\pi^i, \frac{1}{\psi}\beta_c^i, -1\right]$ and $x_t = [x_{\pi,t}, x_{c,t}, x_{d,t}]$.

F.4 Permanent and transitory components

Using the eigenfunction problem of Alvarez and Jermann (2005) and Hansen (2012), we calculate the permanent and transitory components of the SDF. Let $f^{(e)}_{i,t+1}$ be the eigenfunction, then

$$E_t\left[M_{i,t+1}f^{(e)}_{i,t+1}\right] = \xi_i f^{(e)}_{i,t}.$$

The permanent and transitory component are

$$M^P_{i,t+1} = M_{i,t+1}\frac{f^{(e)}_{i,t+1}}{\xi_i f^{(e)}_{i,t}}, \quad M^T_{i,t+1} = \frac{\xi_i f^{(e)}_{i,t}}{f^{(e)}_{i,t+1}},$$

respectively. Conjecture that $f^{(e)}_{i,t+1} = \exp\{f_{i,c}x_{c,t+1} + f_{i,\pi}x_{\pi,t+1} + f_{i,d}x_{d,t+1}\}$. Then

$$\begin{aligned}
& \log E_t\left[M_{i,t+1}f^{(e)}_{i,t+1}\right] \\
= & \log E_t \exp\{m_{i,t+1} + f_{i,c}x_{c,t+1} + f_{i,\pi}x_{\pi,t+1} + f_{i,d}x_{d,t+1}\} \\
= & \log E_t \exp\left\{\bar{m}_i - \frac{1}{\psi}\beta^c_i x_{c,t} - \beta^\pi_i x_{\pi,t} + x_{d,t} + K_{i,4}\varepsilon_{i,t+1} + f_{i,c}\left(\rho_c x_{c,t} + \rho_{c\pi}x_{\pi,t} + \sigma_{x,c}\varepsilon_{c,t+1}\right)\right. \\
& \left. + f_{i,\pi}\left(\rho_\pi x_{\pi,t} + \sigma_{x,\pi}\varepsilon_{\pi,t+1}\right) + f_{i,d}\left(\rho_d x_{d,t} + \sigma_{x,d}\varepsilon_{d,t+1}\right)\right\} \\
= & \bar{m}_i - \left(\frac{1}{\psi}\beta^c_i - f_{i,c}\rho_c\right)x_{c,t} - \left(\beta^\pi_i - f_{i,c}\rho_{c\pi} - f_{i,\pi}\rho_\pi\right)x_{\pi,t} + \left(1 + f_{i,d}\rho_d\right)x_{d,t} + \frac{1}{2}K_{i,5}\Sigma K'_{i,5}
\end{aligned}$$

where

$$\begin{aligned}
K_{i,5} = & \left[-\sigma_\pi, -\gamma\sigma_c, \theta\sigma_d, \left(\frac{\rho_{c\pi}}{1-\kappa_c\rho_\pi}\frac{\frac{1}{\psi}-\gamma}{1-\kappa_c\rho_c}\beta^c_i\kappa^2_c + f_{i,\pi}\right)\sigma_{x,\pi}, \left(\frac{\frac{1}{\psi}-\gamma}{1-\kappa_c\rho_c}\beta^c_i\kappa_c + f_{i,c}\right)\sigma_{x,c}, \ldots\right. \\
& \left. \ldots\left(\frac{(\theta-1)\kappa_c}{1-\kappa_c\rho_d} + f_{i,d}\right)\sigma_{x,d}\right]
\end{aligned}$$

Using the definition

$$\log E_t\left[M_{i,t+1}f^{(e)}_{i,t+1}\right] = \log\xi_i + f_{i,c}x_{c,t} + f_{i,\pi}x_{\pi,t} + f_{i,d}x_{d,t}$$

to match the coefficients, we get:

$$\begin{aligned} f_{i,c} &= \frac{-\frac{1}{\psi}\beta_i^c}{1-\rho_c} \\ f_{i,\pi} &= \frac{-\beta_i^\pi}{1-\rho_\pi} + \frac{\rho_{c\pi}}{1-\rho_\pi}\left(\frac{-\frac{1}{\psi}\beta_i^c}{1-\rho_c}\right) \\ f_{i,d} &= \frac{1}{1-\rho_d} \\ \log\xi_i &= \bar{m}_i + \frac{1}{2}K_{i,5}\Sigma K_{i,5}'. \end{aligned}$$

Thus

$$\begin{aligned} m_{i,t+1}^T &= \log\xi_i + f_{i,c}x_{c,t} + f_{i,\pi}x_{\pi,t} + +f_{i,d}x_{d,t} - f_{i,c}x_{c,t+1} - f_{i,\pi}x_{\pi,t+1} - -f_{i,d}x_{d,t+1} \\ &= \log\xi_i - \frac{1}{\psi}\beta_i^c x_{c,t} - \beta_i^\pi x_{\pi,t} + x_{d,t} - f_{i,c}\sigma_{x,c}\varepsilon_{c,t+1} - f_{i,\pi}\sigma_{x,\pi}\varepsilon_{\pi,t+1} - f_{i,d}\sigma_{x,d}\varepsilon_{d,t+1}, \end{aligned}$$

and the permanent component is

$$\begin{aligned} m_{i,t+1}^P &= m_{i,t+1} - m_{i,t+1}^T \\ &= \bar{m}_i - \log\xi_i + K_{i,4}\varepsilon_{i,t+1} + f_{i,c}\sigma_{x,c}\varepsilon_{c,t+1} + f_{i,\pi}\sigma_{x,\pi}\varepsilon_{\pi,t+1} + f_{i,d}\sigma_{x,d}\varepsilon_{d,t+1} \\ &= \bar{m}_i - \log\xi_i + K_{i,5}\varepsilon_{i,t+1}. \end{aligned}$$

F.5 Infinite maturity yields

Nominal bonds. To obtain the yields of a bond with maturity n, note that

$$m_{i,t+1} = -r_{i,t} - \frac{1}{2}K_{i,4}\Sigma K_{i,4}' + K_{i,4}\varepsilon_{i,t+1}$$

and the risk-free rate is

$$r_{i,t} = \bar{r}_i + \beta_i' \cdot x_t,$$

where $\beta_i = \left[\beta_\pi^i, \frac{1}{\psi}\beta_c^i, -1\right]'$ and $x_t = [x_{\pi,t}, x_{c,t}, x_{d,t}]'$. We can write the law of motion of x_t in matrix form as:

$$x_{t+1} = Kx_t + P\varepsilon_{i,t+1},$$

where

$$K = \begin{bmatrix} \rho_\pi & 0 & 0 \\ \rho_{c\pi} & \rho_c & 0 \\ 0 & 0 & \rho_d \end{bmatrix}, \quad P = \begin{bmatrix} 0 & 0 & 0 & \sigma_{x,\pi} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{x,c} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{x,d} \end{bmatrix}.$$

The yield on an n period maturity bond is:

$$r_{i,t}^{n} = A_i^n + B_i^{n'} \cdot x_t,$$

where

$$\begin{aligned}
A_i^n &= \bar{r}_i - \frac{1}{n}\sum_{j=1}^{n}\left(j \cdot B_i^j\right)' \frac{P\Sigma P'}{2}\left(j \cdot B_i^j\right) + \frac{1}{n}\left(\sum_{j=1}^{n-1} j \cdot B_i^{j'}\right) P\Sigma K'_{i,4} \\
B_i^{n'} &= \frac{1}{n}\beta_i' \sum_{i=0}^{n-1} K^i = \frac{1}{n}\beta_i' \left(I-K\right)^{-1}\left(I-K^n\right).
\end{aligned}$$

Infinite maturity yield. By letting $n \to \infty$, it is possible to show that:

$$\begin{aligned}
\lim_{n\to\infty} B_i^n &= 0 \\
\lim_{n\to\infty} A_i^n &= \bar{r}_i - \beta_i'(I-K)^{-1}P\Sigma\left[\frac{P'}{2}\left[(I-K)^{-1}\right]'\beta_i - K'_{i,4}\right]
\end{aligned}$$

It follows that the yield on the infinite maturity bond is constant and equal to $r_i^{\infty} = \lim_{n\to\infty} A_i^n$. Specifically, we have

$$\lim_{n\to\infty} B_i^n = \lim_{n\to\infty}\frac{1}{n}\beta_i'\left(I-K\right)^{-1}\left(I-K^n\right) = 0,$$

and

$$\begin{aligned}
\lim_{n\to\infty} A_i^n &= \bar{r}_i + \lim_{n\to\infty}\frac{1}{n}\beta_i'\left[I + (I+K) + \ldots + \left(I + K + \cdots + K^{n-2}\right)\right] P\Sigma K'_{i,4} \\
&\quad - \lim_{n\to\infty}\frac{1}{n}\beta_i'\left[I\frac{P\Sigma P'}{2}I + (I+K)\frac{P\Sigma P'}{2}(I+K)' + \ldots\right]\beta_i \\
&= \bar{r}_i + \left(\lim_{n\to\infty}\frac{1}{n}\beta_i'\left[\sum_{i=0}^{n-2}\sum_{j=0}^{i}K^j\right]P\Sigma K'_{i,4}\right) - \left(\beta_i'(I-K)^{-1}\frac{P\Sigma P'}{2}\left[(I-K)^{-1}\right]'\beta_i\right) \\
&= \bar{r}_i + \left(\lim_{n\to\infty}\frac{1}{n}\beta_i'\left[\sum_{i=0}^{n-2}(I-K)^{-1}\left(I-K^{i+1}\right)\right]P\Sigma K'_{i,4}\right) - \left(\beta_i'(I-K)^{-1}\frac{P\Sigma P'}{2}\left[(I-K)^{-1}\right]'\beta_i\right) \\
&= \bar{r}_i + \left(\lim_{n\to\infty}\frac{n-1}{n}\beta_i'\left[(I-K)^{-1}\right]P\Sigma K'_{i,4}\right) - \left(\lim_{n\to\infty}\frac{1}{n}\beta_i'\left[\sum_{i=0}^{n-2}\left(I-K^{i+1}\right)\right]P\Sigma K'_{i,4}\right) \\
&\quad - \left(\beta_i'(I-K)^{-1}\frac{P\Sigma P'}{2}\left[(I-K)^{-1}\right]'\beta_i\right) \\
&= \bar{r}_i + \left(\beta_i'\left[(I-K)^{-1}\right]P\Sigma K'_{i,4}\right) - (0) - \left(\beta_i'(I-K)^{-1}\frac{P\Sigma P'}{2}\left[(I-K)^{-1}\right]'\beta_i\right) \\
&= \bar{r}_i - \beta_i'(I-K)^{-1}P\Sigma\left[\frac{P'}{2}\left[(I-K)^{-1}\right]'\beta_i - K'_{i,4}\right].
\end{aligned}$$